IOSR Journal of XXXXXXXX (IOSRJXXXX)
ISSN : XXXX-XXXX Volume X, Issue X (XXXX-XXXX 2015), PP XX-XX
www.iosrjournals.org

TRAI RASHIQ BARGIA SUTRA AND RANA'S CONSTANT

Author[1]: **MOHAMMAD MAKBUL HOSSAIN (RANA)**

[1]*Department: COURSE TEACHER: APPAREL TECHNOLOGY, PROFESSIONAL INSTITUTE OF SCIENCE AND FASHION TECHNOLOGY (NU), UTTARA, DHAKA, BANGLADESH*
[2]*Department:* As a student of MIT, *INSTITUTE OF INFORMATION TECHNOLOGY, JAHANGIRNAGAR UNVERSITY, SAVAR, DHAKA, BANGLADESH*

Abstract : ***Like as,***

1. ***Polidromik Number: (111, 222, 121, 525 its value same as from left to right starting number setup)***
2. ***Horzat Number: (the number is divided by the sum of groups all number in, like in 12 sums (1+2) 3 is the Hozrat number of 12.)***
3. ***Demloa Number: (214423, 21+23 =44 that of the sum 1st +last number =middle number)***
4. ***UD Number : (ups and down number like 6, 9, 81, 18)***
5. ***Kaprekar Constant: (the professor kaprekar in 1946 inventing the constant number 6174)***
6. ***Kaprekar number: (2025 , cut in middle and ad 20+25=45 square then same previous number 2025)***

In this Connection,

Axioms defined Mohammad Makbul Hossain Rana using sequential, odd and even numbers, "TRAI RASHIQ BARGIA SUTRA & RANA'S CONSTANT", is a new invention and it is an extra ordinary and in-depth development in mathematics. His profound achievements in special types of Number Theory. In this theory, he establishes a particular relation between three consecutive integer/ odd/even numbers which is called "Rana's Trai Rashiq bargio sutra & Constant as odd and even 8 and integer 2 up to nth term.

Keywords : -

I. INTRODUCTION

Rana's Trai Rashiq bargio sutra & Constant, as odd and even "R=8" and interger"R= 2" up to nth term. ***Provided extraordinarily deep theorems that laid the foundation for the complete classification of finite simple numbers, one of the greatest achievements of twentieth century in mathematics like, professor Kaprekar constant. Simple numbers are atoms from which all finite numbers are built have a common relation. In a major breakthrough, proved that every number (integer/even/odd) have a common number of elements. Later extended this result to establish a common constant of an important kind of finite simple number called an "R=2" for integer & R=8, for odd /even number. At this point, the classification project came within incredible conclusion that all finite simple number belongs to certain standard families. In-depth and influential. His complements each other's and together forms the backbone of modern number theory.***

II. HEADING S

Name of theory: Trai Rashiq Bargia Sutra and Rana's Constant

"Par par tinty aungker Barger auntor dhoer auntor akti dhrobo sonkha."

[Bangla Version: ci ci wZbwU As‡Ki e‡Mi A šiØ‡qi A ši GKwU ae msL¨v , msL¨wU = 2]

[Bangla Version: ci ci wZbwU †Rvo ev †e‡Rvo msL¨vi e‡Mi A šiØ‡qi A ši GKwU ae msL¨v, msL¨wU = 8]
ivbvi aeK =2,8 Rana's dhrobok Integer number = 2, Even/odd (zore/bizore) number = 8

III. INDENTATIONS AND EQUATIONS

<u>Theses/theory</u>:
1.1. Name of theory: Trai Rashiq Bargia Sutra and Rana's Constant.

1.1.1 Basic: Number theory.

1.1.2. Definition/sutra: Par par tinty aungker Barger auntor dhoer auntor akti dhrobo sonkha.

1.1.3. [Bangla Version: ci ci wZbwU As‡Ki e‡Mi A šiØ‡qi A ši GKwU ae msL¨v , msL¨wU = 2]

1.1.4. [Bangla Version: ci ci wZbwU †Rvo ev †e‡Rvo msL¨vi e‡Mi A šiØ‡qi A ši GKwU ae msL¨v, msL¨wU = 8]
1.1.5. ivbvi aeK=2, Rana's dhrobok Integer number = 2, Even/odd (zore/bizore) number = 8

1.1.6. Constant: integer number "R=2", Even/odd number "R= 8"

Proof this theory and constant "R=2 "/ "8":

1.1.7.

1.2.1. As 1, 2, 3 are three integer number

Proof:

$1^2 = 1$

] = 3

$2^2 = 4$]= 2 [The Rana's constant for integer Number up to n term]

] = 5

$3^2 = 9$

2.3. As 1, 3, 5 are three odd number

$1^2 = 1$

]= 8

$3^2 = 9$]= 8 [The Rana's constant for ODD Numb up to n term]

]= 16

$5^2 = 25$

1.2.2. As 2, 4, 6 are three even number

$2^2 = 4$

]= 12

$4^2 = 16$]= 8 [The Rana's constant for EVEN Numb up to n term]

]= 20

$6^2 = 36$

1.2.3. Example: let Three number are (a – 1), a, (a + 1)
According to Rana's Trai Rashiq Bargia Sutra

Proof:

When, a = 1, 2, 3………..n [integer number]

$\{(a + 1)^2 – a^2\} – \{ a^2 – (a – 1)\}^2$

$= \{a^2 + 2.a.1 + 1 - a^2\} – \{ a^2 – a^2 + 2.a.1 – 1^2\}$

$= \{2.a.1 + 1 \} – \{ 2.a.1 – 1\}$

$= \{2.a.1 + 1 – 2.a.1 + 1\}$

$= \{1 + 1\}$

$=2$

This the Rana's Constant "R=2" or "R"

1.2.4. Example: let Three number are a, (a+2), (a +4)
According to Rana's Trai Rashiq Bargia Sutra

Proof:

a, (a + 2), (a + 4)

When a= 2, 4, 6……..n [EVEN Number]

$\{ a^2 - (a + 2)^2\} – \{ (a+2)^2 – (a + 4)^2\}$

[$\{ 2^2 - (2 + 2)^2\} – \{(2+2)^2 – (2+ 4)^2\} = 8$ when a =2]

[$\{ 4^2 - (4 + 2)^2\} – \{(4+2)^2 – (4+ 4)^2\} = 8$ when a =4]

[$\{ 6^2 - (6 + 2)^2\} – \{(6+2)^2 – (6+ 4)^2\} = 8$ when a =6]

$= \{a^2 - a^2 - 2.a.2 - 4\} – \{ a^2 + 2.a.2 + 4 – (a^2 + 2.a.4 + 16)\}$

$= \{-2.a.2 - 4\} – \{ a^2 + 2.a.2 + 4 – a^2 - 2.a.4 - 16\}$

$= \{-4.a - 4\} – \{ 2.a.2 + 4 – 2.a.4 - 16\}$

$= \{-4.a - 4 – 4.a - 4 + 8.a + 16\}$

$= \{-4.a - 4 – 4.a - 4 + 8.a + 16\}$

$=\{- 8 + 16\}$

$=8$

This the Rana's Constant "R=8" or "R"

1.2.5. Example: let Three number are a, (a+2), (a +4)
According to Rana's Trai Rashiq Bargia Sutra

Proof:

a , (a+2), (a + 4)

When, a= 1,3,5………..n [ODD number]

$\{ a^2 - (a + 2)^2\} – \{ (a+2)^2 – (a + 4)^2\}$

$[\{ 1^2 - (1 + 2)^2\} – \{(1+2)^2 – (1 + 4)^2\} = 8$ **when** $a =1]$
$[\{ 3^2 - (3 + 2)^2\} – \{(3+2)^2 – (3 + 4)^2\} = 8$ **when** $a =3]$
$[\{ 5^2 - (5 + 2)^2\} – \{(5+2)^2 – (5 + 4)^2\} = 8$ **when** $a =5]$

$= \{a^2 - a^2 - 2.a.2 - 4 \} – \{ a^2 +2.a.2 + 4 – (a^2 +2.a.4 + 16)\}$
$= \{-2.a.2 - 4\} – \{ a^2 +2.a.2 + 4 – a^2 - 2.a.4 - 16\}$
$= \{-4.a - 4\} – \{ 4.a + 4 – 8.a - 16\}$
$= \{-4.a - 4 – 4.a - 4 + 8.a + 16\}$
$= \{-4.a - 4 – 4.a - 4 + 8.a + 16\}$
$=\{- 8 + 16\}$
$=\{- 8 + 16\}$
$=8$
This the Rana's Constant "8" or "R"

IV. FIGURES AND TABLES

2.1. Definition: Par par tinty aungker barger antor dhoer auntor akti dhrobo sonkha.

Exercise: - 1

(Numerical Problems)

2.1.1) Sum of any three integers square value as 35 and multiple value of 1st & 3rd number as 5 and Rana's constant "8" , proof Rana's theory and determine the value of three numbers.

†h †Kvb wZbwU µwgK msL¨vi e‡Mi mgwó 35 Ges 1g I 3q c‡`i ¸b dj 5 Ges ivbvi aeK= 8 n‡j msL¨v wZbwU wbbq Ki |

2.1.2) Determine the value of three numbers that the Rana's constant as "8" and sum of square is 56 and 1st & 3rd number multiple is 12.

hw` ivbvi aeK= 8 nq Ges e‡Mi mgwó 56 , 1g I 3q c‡`i ¸b dj 12 nq Z‡e msL¨v wZbwU wbbq Ki |

2.1.3) If R=8 and the square sum value of three number is 83 and 1st & 3rd number multiple is 21. Determine the number odd or even.

hw` R= 8 Ges wZbwU msL¨vi e‡Mi mgwó 83 , 1g I 3q c‡`i ¸b dj 21 nq Z‡e msL¨v wZbwU ‡Rvo bv †e‡Rvo wbbq Ki |

2.1.4) If R=2 and the square sum value of three number is 14 and 1st & 3rd number multiple is 3.determine the value of those number.

hw` R= 2 Ges msL¨v wZbwUi e‡Mi mgwó 14 , 1g I 3q c‡`i ¸b dj 3, nq Z‡e msL¨v wZbwU wbbq Ki |

2.1.5) Determine the value of a, b, c when a2 + b2 + c2 =29 & ac=8 and constant R=2.

hw` R=2. nq Ges a2 + b2 + c2 =29 & ac=8 nq, Z‡e a, b, c Gi gvb wbbq Ki |

g š Z e¨(Comments) :
+8801911133012,+8801199042799
e-mail: atex.rana@gmail.com, mhrana01@yahoo.com

Solution: 2.1.1)

Let, a, b, c is three integers, According to the question as
$a^2 + b^2 + c^2 = 35$ -------(i),
a x c = 5 ----------------(ii),
a = 5/c ------------------------------(iii)

Constant R = 8
According to the rana's theory.
$\{a^2 - b^2\} - \{ b^2 - c^2\} = 8$
=> $a^2 - 2b^2 + c^2 = 8$
=> $a^2 + b^2 + c^2 = 8 + 3b^2$
=>35 = 8 + 3b2
=> 35 - 8 = 3b2
=> 27 = 3b2
=> $b^2 = 9$
=> b = + 3, -3
from eqn (i)
$a^2 + b^2 + c^2 = 35$ --------(i),
=> $a^2 + 3^2 + c^2 = 35$
=> $a^2 + c^2 = 35 - 9$ (put the value of b)
=> $a^2 + c^2 = 26$ ----(iv)
From eqn (iv) & (iii)
=> $a^2 + c^2 = 26$
=> $(5/c)^2 + c^2 = 26$
=> $25/c^2 + c^2 = 26$
=> $25/c^2 + c^2 = 26$
=> $(25 +c^4)/c^2= 26$
=> $25 +c^4 = 26$ x c2
=> $c^4 - 26c^2 +25 =0$
=> $c^4 - 25c^2- c^2+25 =0$
=> $c^2(c^2 - 25)-1(c^2-25) =0$
=> $(c^2 - 25)(c^2-1) =0$
=> $(c^2 - 25) =0$
=> $c^2=1$
=> c =1, -1
or
=> $(c^2-25) =0$
=> $c^2 =25$
=> c = 5, -5
From eqn (iii)
a=5/c
a=5/5=1 [c = 5]
a = 5/1 =5 [c = 1]

The values three numbers as 1, 3, 5 or 5, 3, 1 (Odd)

Solution: 2.1.2)
Let, p, q, r is three integers, According to the question as
$p^2 + q^2 + r^2 = 56$ ---------(i),
$p \times r = 12$ ----------------(ii),
$p = 12/r$ ---------------------------------(iii)
Constant R = 8
According to the rana's theory.
$\{p^2 - q^2\} - \{q^2 - r^2\} = 8$
$\Rightarrow p^2 - 2q^2 + r^2 = 8$
$\Rightarrow p^2 + q^2 + r^2 = 8 + 3q^2$
$\Rightarrow 56 = 8 + 3q^2$
$\Rightarrow 56 - 8 = 3q^2$
$\Rightarrow 48 = 3q^2$
$\Rightarrow q^2 = 16$
$\Rightarrow q = +4, -4$
From eqn (i)
$p^2 + q^2 + r^2 = 56$
$\Rightarrow p^2 + 4^2 + r^2 = 56$
$\Rightarrow p^2 + r^2 = 56 - 16$
$\Rightarrow p^2 + r^2 = 40$ ----(iv)
From eqn (iv) & (iii)
$\Rightarrow p^2 + r^2 = 40$
$\Rightarrow (12/r)^2 + r^2 = 40$
$\Rightarrow 144/r^2 + r^2 = 40$
$\Rightarrow 144/r^2 + r^2 = 40$
$\Rightarrow (144 + r^4)/r^2 = 40$
$\Rightarrow 144 + r^4 = 40 \times r2$
$\Rightarrow r^4 - 40r^2 + 144 = 0$
$\Rightarrow r^4 - 36r^2 - 4r^2 + 144 = 0$
$\Rightarrow r^2(r^2 - 36) - 4(r^2 - 36) = 0$
$\Rightarrow (r^2 - 36)(r^2 - 4) = 0$
$\Rightarrow (r^2 - 36) = 0$
$\Rightarrow r^2 = 36$
$\Rightarrow r = 6, -6$
or
$\Rightarrow (r^2 - 4) = 0$
$\Rightarrow r^2 = 4$
$\Rightarrow r = 2, -2$
From eqn (iii)
$P = 12/r$
$P = 12/6 = 2$ [r = 6]
$p = 12/2 = 6$ [r = 2]
The values three numbers as 2, 4, 6 or 6, 4, 2 or -2,-4,-6, (even)

Solution: 2.1.3)
Let, p, q, r is three integers, According to the question as
$x^2 + y^2 + z^2 = 83$ ---------(i),
$x \times z = 21$ ----------------(ii),
$x = 21/z$ ---------------------------------(iii)
Constant R = 8
According to the rana's theory.
$\{x^2 - y^2\} - \{y^2 - z^2\} = 8$

=> $x^2 - 2y^2 + z^2 = 8$
=> $x^2 + y^2 + z^2 = 8 + 3y^2$
=> $83 = 8 + 3y^2$
=> $83 - 8 = 3y^2$
=> $75 = 3y^2$
=> $y^2 = 25$
=> $y = + 5, -5$
From eqn (i)
$x^2 + y^2 + z^2 = 83$
=> $x^2 + 5^2 + z^2 = 83$
=> $x^2 + z^2 = 83 – 25$
=> $x^2 + z^2 = 58$ ----(iv)
From eqn (iv) & (iii)
=> $x^2 + z^2 = 58$
=> $(21/z)^2 + z^2 = 58$
=> $441/z^2 + z^2 = 58$
=> $(441 + z^4)/z^2 = 58$
=> $441 + z^4 = 58z^2$
=> $441 - 58z^2 + z^4 = 0$
=> $49(9 - z^2) - z^2 (9- z^2) = 0$
=> $(49 - z^2)(9 - z^2) = 0$
=> $49 =z^2$ or $9 = z^2$
=> $-7,7 = z$ or $-3,3 = z$
From eqn (iii)
$x =21/z$
$x=21/7 = 3$ [$z = 7$]
$x = 21/3 =7$ [$z = 3$]
The values three numbers as 3, 5, 7 or 7, 5, 3 or -3,-5,-7(odd), Proved

Solution: 2.1.4)
Let, l, m, n is three integers, According to the question as
$l^2 + m^2 + n^2 = 14$---------(i),
$l \times n = 3$ ------------------(ii),
$l = 3/n$ ------------------------------- -(iii)
Constant R = 2
According to the rana's theory.
$\{l^2 - m^2\} – \{ m^2 – n^2\} = 2$
=> $l^2 - 2m^2 + n^2 = 2$
=> $l^2 + m^2 + n^2 = 2 + 3m^2$
=> $14 = 2 + 3m^2$
=> $14 - 2 = 3m^2$
=> $12 = 3m^2$
=> $m^2 = 4$
=> $m = + 2, -2$
from eqn (i)
$l^2 + m^2 + n^2 = 14$
=> $l^2 + 2^2 + n^2 = 14$
=> $l^2 + n^2 = 14 – 4$
=> $l^2 + n^2 = 10$ ----(iv)
From eqn (iv) & (iii)
=> $l^2 + n^2 = 10$
=> $(3/n)^2 + n2 = 10$
=> $9/n^2 + n^2 = 10$
=> $(9 + n^4)/n^2 = 10$

=> $9 + n^4 = 10n^2$
=> $9 – 10n^2 + n^4 = 0$
=> $9 – n^2 -9n^2 + n^4 = 0$
=> $1(9 - n^2) - n^2 (9- n^2) = 0$
=> $(9 - n^2)(1 - n^2) = 0$
=> $9 = n^2$ or $1 = n^2$
=> -3,3 = n or -1,1 = n
From eqn (iii)
l =3/n
l=3/1 = 3 [n = 1]
l = 3/3 =1 [n = 3]
The values three numbers as 1, 2, 3 or 3, 2,1 or -1,-2,-3 (integer) Proved

Solution: 2.1.5)
Let, a, b, c is three integers
According to the question
$a^2 + b^2 + c^2 =29$--------(i)
ac = 8 ------------------(ii)
a=8/c ------------------(iii)
constant R=2.
According to the rana's theory.
$\{a^2 - b^2\} – \{ b^2 – c^2\} = 2$
=> $a^2 - 2b^2 + c^2 = 2$
=> $a^2 + b^2 + c^2 = 2 + 3b^2$
=> $29 = 2 + 3b^2$
=> $29 - 2 = 3b^2$
=> $27 = 3b^2$
=> $b^2 = 9$
=> b = + 3, -3
from eqn (i)
$a^2 + b^2 + c^2 = 29$
=> $a^2 + 3^2 + c^2 = 29$
=> $a^2 + c^2 = 29 – 9$
=> $a^2 + c^2 = 20$ ----(iv)
From eqn (iv) & (iii)
=> $a^2 + c^2 = 20$
=> $(8/c)^2 + c^2 = 20$
=> $64/c^2 + c^2 = 20$
=> $64/c^2 + c^2 = 20$
=> $(64 +c4)/c^2= 20$
=> $64 +c4 = 20 \times c^2$
=> $c4 - 20c^2 +64 =0$
=> $c4 - 16c^2- 4c^2+64 =0$
=> $c2(c2 - 16)-4(c^2-16) =0$
=> $(c^2 - 16)(c^2-4) =0$
=> (c - 16) =0
=> $(c^2-4)=0$
=> c =2, -2
or
=>$(c^2-16) =0$
=> $c^2 =16$
=> c = 4, -4
From eqn (iii)
a=8/c

a=8/2=4 [c = 2]
a = 8/4 =2 [c = 4]
The values three numbers as 2, 3, 4 or 4, 3, 2 (integer), Proved

V. CONCLUSION

At this point, the classification project came within. Its almost incredible conclusion that all finite simple number belongs to certain standard families. The achievements of Mohammad Makbul Hossain (Rana)**is the extraordinary, in-depth and influential. His complements each other's and together forms the backbone of modern number theory**

Learning this theory the mathematics student will benefited like the followings:

1. **Power of Math in a single equation.**
2. **Odd and Even Number**
3. **Relation between 1 to n-1 number (Odd Number)**
4. **Relation Between 1 to n+1 number (even number)**
5. **Relation between 1 to n number (integer number)**
6. **Negative and positive numbers in a same relation.**
7. **Infinity Relation for number.**
8. **Common relation from all number in integer and odd, even number.**
9. **It is a "Trai Rashiq" equation so it needs two values to determine the third value.**

Acknowledgements

I gratefully acknowledge my indebtedness to Late Rias uddin (Teacher, Dhanua sarkary Primary school), Late Abdul Matin (Teacher, Mowna High School) Md. Afzal Hossain and Abul Hasem (Asst./Professor, Bhawal Badra alam Govt. University College) who have acted as my guideline/supervisor. They inspire me to introduce to the research step to step" in Global context give some extra ordinary invention in the course of my life. In different step of my study they give proper education and made me confident that I can carry on further research as ferric alarm from waste iron, natural color cotton, ak goror langol, number theory in this research work, also Mr. Fazlul Karim patwary (Director IIT, Jahangirnagar University), Dr. Zakir Hossain & Dr. Iyub Nabi (Directory Primeasia University), Md. Ismail Hossain (Teacher, sher–e-banglanager govt, boys high school) Dr. Monibur Chowdhury and Mr. Seriazul Islam (Prof/ head department Math Dept, Dhaka University) help and guidance have been an outstanding factor for the completion of this work.

I take this opportunity to express my heart full thanks to all of the Authority of IOSRJ for providing me all sorts of valuable information. And Professional institute of Sciences and fashion Technology.

Finally, I extend my indebtedness to my wife Dr. Mariam Akter, friends, office colique, my beloved parents in the form of financial support, encouragement and advice, it can never be returned adequately and these support and blessing have made it possible to carry out this and continuing research work to completion

REFERENCES

[1] 17th science and technology fair 1993 "Number playing by mathematics" DT: 7-9 Th December 1993
[2] Copyright Registration No. 10482 COPR DT: 31/08/2008
[3] Theory of number
[4]Real number analysis as Sequence of number as . **Number**

1^2 = 1	**28^2 = 784**
4^2 = 16	**29^2 = 841**
5^2 = 25	**30^2 = 900**
…….	
25^2 = 625	**50^2 = 2500**

n…………..….n^2

[5] *Shown / Approved by Member secretary of Bangladesh mathematics society and chairman, math department Dhaka university.*

Proposal for the research work to publish in IOSRJ

SUBMITTED TO:
Editorial Board,
IOSR Journals
International Organization of Scientific Research (IOSR)

SUBMITTED BY:
MOHAMMAD MAKBUL HOSSAIN (RANA)
Course Teacher
Professional Institute of Sciences Fashion Technology
House#10, Road # 3/c, Sector# 09, Uttara Model Town
Dhaka-Bangladesh-1230
Cell: +8801199042799
E-mail. atex.rana@gmail.com
phd.rana11@gmail.com
www.Matherana.syntahsite.com

SUBMITTED DATE:
26/06/2015

In Number binnash

1^2	=	1	26^2	=	676
2^2	=	4	27^2	=	729
3^2	=	9	28^2	=	784
4^2	=	16	29^2	=	841
5^2	=	25	30^2	=	900
6^2	=	36	31^2	=	961
7^2	=	49	32^2	=	1024
8^2	=	64	33^2	=	1089
9^2	=	81	34^2	=	1156
10^2	=	100	35^2	=	1225
11^2	=	121	36^2	=	1296
12^2	=	144	37^2	=	1369
13^2	=	169	38^2	=	1444
14^2	=	196	39^2	=	1521
15^2	=	225	40^2	=	1600
16^2	=	256	41^2	=	1681
17^2	=	289	42^2	=	1764
18^2	=	324	43^2	=	1849
19^2	=	361	44^2	=	1936
20^2	=	400	45^2	=	2025
21^2	=	441	46^2	=	2116
22^2	=	484	47^2	=	2209
23^2	=	529	48^2	=	2304
24^2	=	576	49^2	=	2401
25^2	=	625	50^2	=	2500

....

....

....

n................................n^2

Young Scientist M.H.Rana, www.matherana.synthasite.com

24^2 = 576 **49^2 = 2401**

25^2 = 625 **50^2 = 2500**

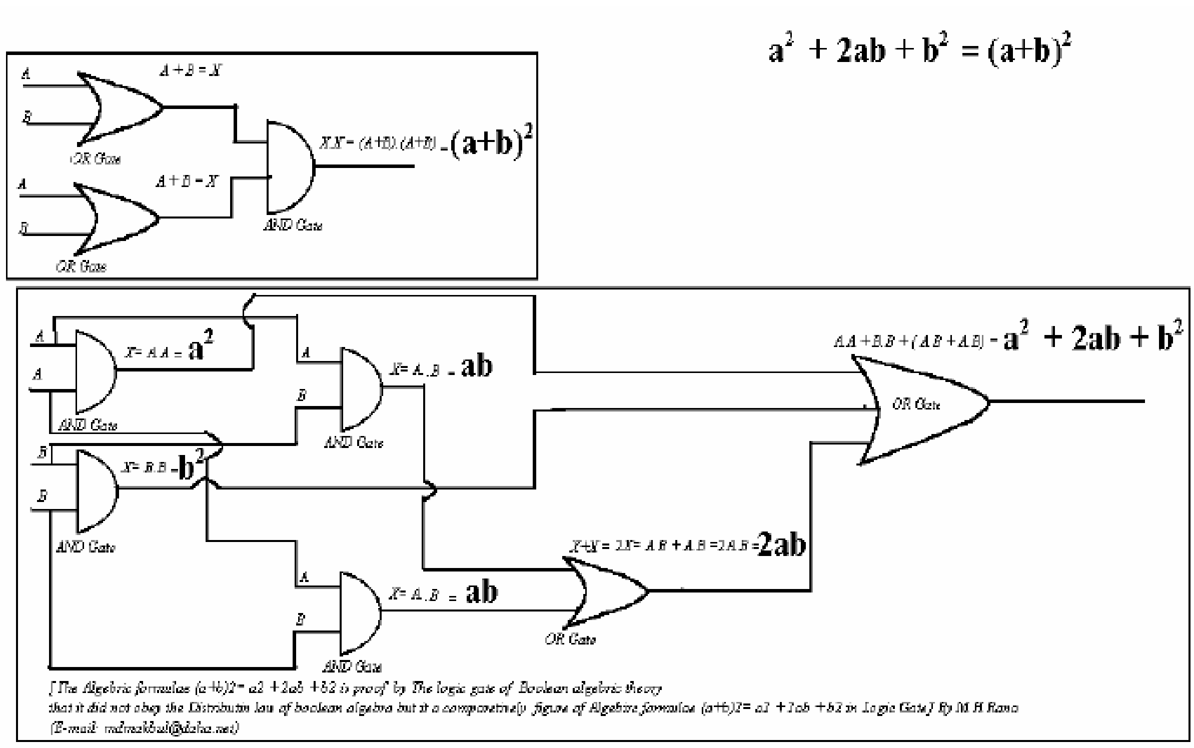
....

....

....

n..................................n^2

Trai Rashik Bargia Sotra By M.H.Rana



Here the algebraic theory

$(a + b)^2 = a^2 + 2ab + b^2$

Proof the Boolean algebra with Logic gate

We can set the theory $(a + b)^2$ by logic gate AND, OR, NOR etc.

This type of simplification helps to the student comparable study between different systems of mathematics. I am also discover another theory in mathematics on Number theory

Ok

Thanks

M.H. Rana

 Young Scientist M.H.Rana, www.matherana.synthasite.com

Algebra

1) **Math Algebra** Basic Identities:

Closure Property of Addition

Sum (or difference) of 2 real numbers equals a real number

Additive Identity

a + 0 = a

Additive Inverse

a + (-a) = 0

Associative of Addition

(a + b) + c = a + (b + c)

Commutative of Addition

a + b = b + a

Definition of Subtraction

a - b = a + (-b)

Closure Property of Multiplication

Product (or quotient if denominator ≠0) of 2 reals equals a real number

Multiplicative Identity

a * 1 = a

Multiplicative Inverse

a * (1/a) = 1 (a ≠ 0)

(Multiplication times 0)

a * 0 = 0

Associative of Multiplication

(a * b) * c = a * (b * c)

Commutative of Multiplication

a * b = b * a

Distributive Law

a(b + c) = ab + ac

Definition of Division

a / b = a(1/b)

2) **Math Algebra Exponents Identities:**

Powers

$x^a x^b = x^{(a+b)}$

$x^a y^a = (xy)^a$

$(x^a)^b = x^{(ab)}$

$x^{(a/b)}$ = b^{th} root of (x^a) = $(b^{th} \sqrt{}(x))^a$

$x^{(-a)} = 1 / x^a$

$x^{(a-b)} = x^a / x^b$

Logarithms

$y = \log_b(x)$ if and only if $x = b^y$

$\log_b(1) = 0$

$\log_b(b) = 1$

$\log_b(x*y) = \log_b(x) + \log_b(y)$

$\log_b(x/y) = \log_b(x) - \log_b(y)$

$\log_b(x^n) = n \log_b(x)$

$\log_b(x) = \log_b(c) * \log_c(x) = \log_c(x) / \log_c(b)$

3) **Math** **Algebra** **Polynomials Identities:**

$(a+b)^2 = a^2 + 2ab + b^2$

$(a+b)(c+d) = ac + ad + bc + bd$

$a^2 - b^2 = (a+b)(a-b)$ (Difference of squares)

$a^3 \pm b^3 = (a \pm b)(a^2 \mp ab + b^2)$ (Sum and Difference of Cubes)

$x^2 + (a+b)x + AB = (x + a)(x + b)$

if $ax^2 + bx + c = 0$ then $x = (-b \pm\sqrt{}(b^2 - 4ac)) / 2a$ (Quadratic Formula)

4) **Math** | **Algebra** | **Functions Identities:**

Synonyms: correspondence, mapping, transformation

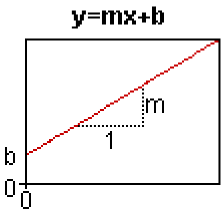
Definition: A function is a **relation** from a domain set to a range set, where each element of the domain set is related to exactly one element of the range set.

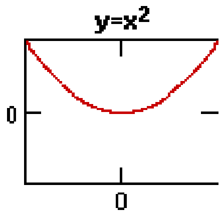
An equivalent definition: A function (f) is a relation from a set A to a set B (denoted f: A�B), such that for each element in the domain of A (Dom(A)), the f-relative set of A (f(A)) contains exactly one element.

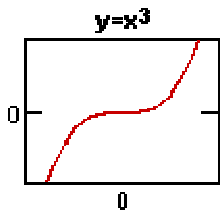
Some common functions (with discussions)

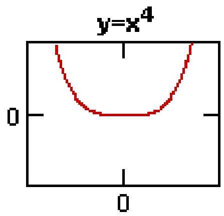
- trig functions
    - sine, sin(x)
    - cosine, cos(x)

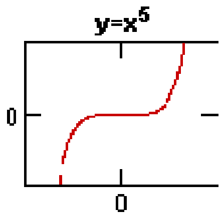
5) **Math** | **Miscellaneous** | **Algebra graphics**

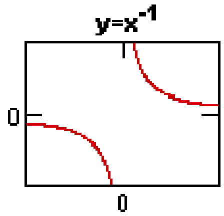


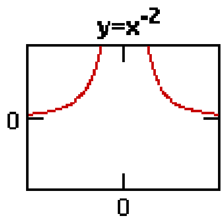


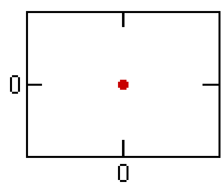
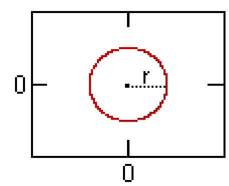
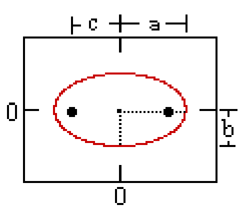
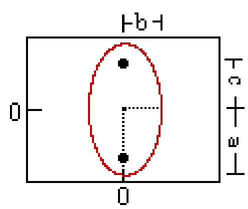
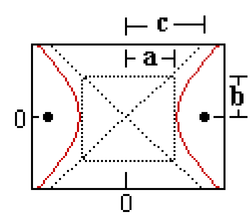


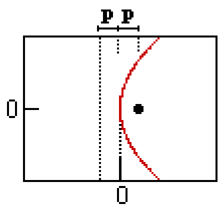
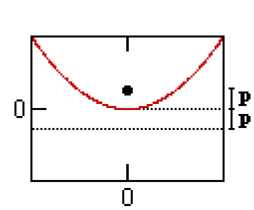
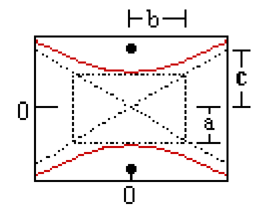








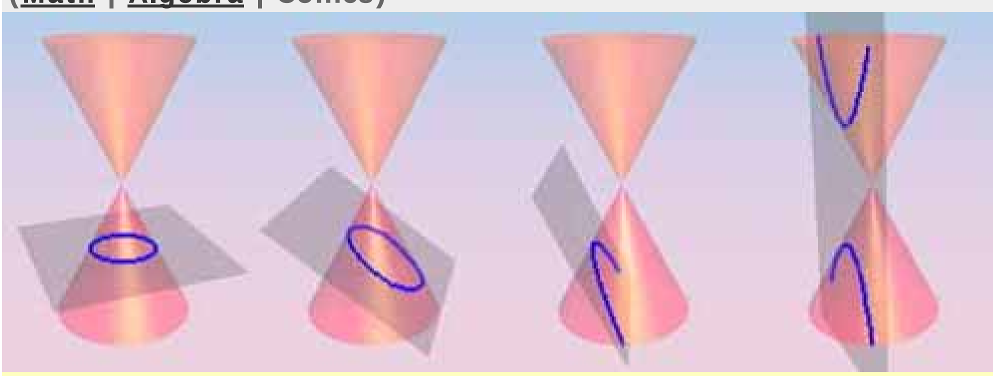
Conic Sections (see also Conic Sections)	**Point**  $x^2 + y^2 = 0$	**Circle**  $x^2 + y^2 = r^2$
Ellipse  $x^2 / a^2 + y^2 / b^2 = 1$	**Ellipse**  $x^2 / b^2 + y^2 / a^2 = 1$	**Hyperbola**  $x^2 / a^2 - y^2 / b^2 = 1$
Parabola	**Parabola**	**Hyperbola**

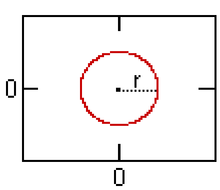
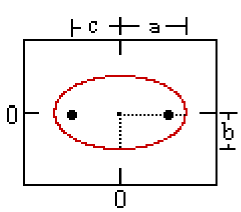
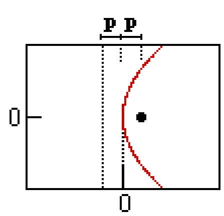
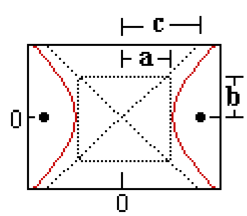
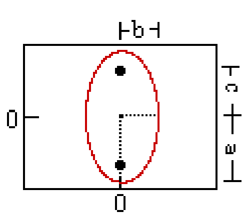
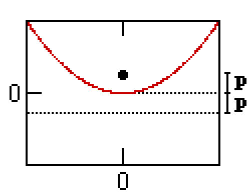
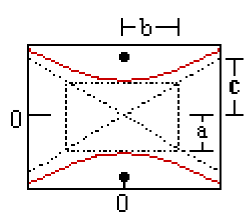
 $4px = y^2$	 $4py = x^2$	 $y^2 / a^2 - x^2 / b^2 = 1$
For any of the above with a center at (j, k) instead of (0,0), replace each x term with (x-j) and each y term with (y-k) to get the desired equation.		

6) **Math** | **Algebra** | **Conics sections**

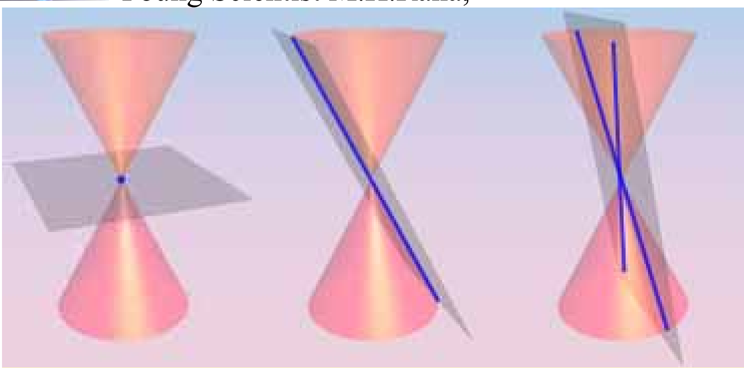
Conic Sections

(**Math** | **Algebra** | **Conics**)

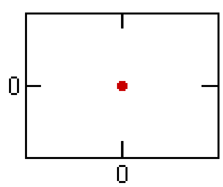


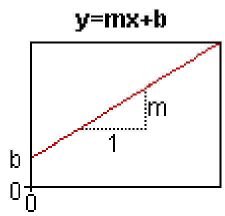
Circle	Ellipse (h)	Parabola (h)	Hyperbola (h)
			
	Ellipse (v)	Parabola (v)	Hyperbola (v)
Definition: A conic section is the intersection of a plane and a cone.			

By changing the angle and location of intersection, we can produce a circle, ellipse, Parabola or hyperbola; or in the special case when the plane touches the vertex: a point, line or 2 intersecting lines.



Point Line Double Line





The General Equation for a Conic Section:

$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$

The type of section can be found from the sign of: $B^2 - 4AC$

If $B^2 - 4AC$ is...	then the curve is a...
< 0	ellipse, circle, point or no curve.
$= 0$	parabola, 2 parallel lines, 1 line or no curve.
> 0	hyperbola or 2 intersecting lines.

The Conic Sections. For any of the below with a center (j, k) instead of (0, 0), replace each x term with (x-j) and each y term with (y-k).

	Circle	Ellipse	Parabola	Hyperbola
Equation (horiz. vertex):	$x^2 + y^2 = r^2$	$x^2 / a^2 + y^2 / b^2 = 1$	$4px = y^2$	$x^2 / a^2 - y^2 / b^2 = 1$
Equations of Asymptotes:				$y = \pm (b/a)x$
Equation (vert. vertex):	$x^2 + y^2 = r^2$	$y^2 / a^2 + x^2 / b^2 = 1$	$4py = x^2$	$y^2 / a^2 - x^2 / b^2 = 1$
Equations of Asymptotes:				$x = \pm (b/a)y$
Variables:	r = circle radius	a = major radius (= 1/2 length major axis) b = minor radius (=	p = distance from vertex to focus (or directrix)	a = 1/2 length major axis b = 1/2 length minor axis c = distance center to focus

		1/2 length minor axis) c = distance center to focus		
Eccentricity:	0	c/a	1	c/a
Relation to Focus:	p = 0	$a^2 - b^2 = c^2$	p = p	$a^2 + b^2 = c^2$
Definition: is the locus of all points which meet the condition...	distance to the origin is constant	sum of distances to each focus is constant	distance to focus = distance to directrix	difference between distances to each foci is constant
Related Topics:	Geometry section on Circles			

7) **Math** | **Miscellaneous** | **Complexity**

Basic Operations

$i = \sqrt{(-1)}$

$i^2 = -1$

[[Rationale that $i^2 = -1$

We know that by definition
$i = \sqrt{(-1)}$
Therefore,
$i^2 = [\sqrt{(-1)}]^2 = -1$.]]

$1 / i = -i$

[[Rationale that $1/i = -i$

We know that by definition
$i = \sqrt{(-1)}$
Similarly,
$i*i = \sqrt{(-1)} * \sqrt{(-1)} = \sqrt{(-1)}^2 = -1$

By algebra we get:

$i*i = -1$

-i*i = 1 *(multiply both sides by -1)*
-i = 1/i *(divide both sides by i)*]]

$i^{4k} = 1;\ i^{(4k+1)} = i;\ i^{(4k+2)} = -1;\ i^{(4k+3)} = -i$ (k = integer)

$\sqrt{(i)} = \sqrt{(1/2)} + \sqrt{(1/2)}\, i$

[[Rationale that $\sqrt{(i)} = \sqrt{(1/2)} + i\sqrt{(1/2)}$

Assuming that
$\sqrt{(i)} = \sqrt{(1/2)} + i\sqrt{(1/2)}$
we can square both sides to get
$i = [\sqrt{(1/2)} + i\sqrt{(1/2)}]^2$
$i = [(1/2) + 2(1/2)\, i + (1/2)\, i^2]$
$i = [(1/2) + i + (1/2)(-1)]$
$i = i$ (which is a true statement)

This is not a proof, but simply evidence that the formula is correct.]]

Complex Definitions of Functions and Operations

$(a + bi) + (c + di) = (a+c) + (b + d)\, i$

$(a + BI)(c + DI) = ac + adi + bci + bdi^2 = (ac - bd) + (ad + bc)\, i$

$1/(a + BI) = a/(a^2 + b^2) - b/(a^2 + b^2)\, i$

$(a + BI) / (c + DI) = (ac + BD)/(c^2 + d^2) + (BC - ad)/(c^2 + d^2)\, i$

$a^2 + b^2 = (a + BI)(a - BI)$ (sum of squares)

$e^{(i\theta)} = \cos\theta + i \sin\theta$

[[Justifications that $e^{i\theta} = \cos(\theta) + i \sin(\theta)$

$e^{ix} = \text{COs}(x) + i \sin(x)$

Justification #1: from the derivative

Consider the function on the right hand side (RHS)

$f(x) = COs(x) + i \sin(x)$

Differentiate this function

$f'(x) = -\sin(x) + i\, COs(x) = i f(x)$

So, this function has the property that its derivative is i times the original function.
What other type of function has this property?

A function g(x) will have this property if

$dg / dx = i g$

This is a differential equation that can be solved with separation of variables

$(1/g)\, dg = i\, dx$

$\int (1/g)\, dg = \int i\, dx$

$\ln| g | = i x + C$

$| g | = e^{i x + C} = e^C e^{i x}$

$| g | = C_2 e^{i x}$

$g = C_3 e^{i x}$

So we need to determine what value (if any) of the constant C_3 makes g(x) = f(x).
If we set x=0 and evaluate f(x) and g(x), we get

$f(x) = COs(0) + i \sin(0) = 1$

$g(x) = C_3 e^{i 0} = C_3$

These functions are equal when $C_3 = 1$.

Therefore,

$COs(x) + i \sin(x) = e^{i x}$

Justification #2: the series method

(This is the usual justification given in textbooks.)

By use of Taylor's Theorem, we can show the following to be true for all real numbers:

sin x $= x - x^3/3! + x^5/5! - x^7/7! + x^9/9! - x^{11}/11! + \dots$

COs x $= 1 - x^2/2! + x^4/4! - x^6/6! + x^8/8! - x^{10}/10! + \dots$

$\mathbf{e^x} = 1 + x + x^2/2! + x^3/3! + x^4/4! + x^5/5! + x^6/6! + x^7/7! + x^8/8! + x^9/9! + x^{10}/10! + x^{11}/11! + \dots$

Knowing that, we have a mechanism to determine the value of $e^{\theta i}$, because we can express it in terms of the above series:

$e\hat{}(\theta i) = 1 + (\theta i) + (\theta i)^2/2! + (\theta i)^3/3! + (\theta i)^4/4! + (\theta i)^5/5! + (\theta i)^6/6! + (\theta i)^7/7! + (\theta i)^8/8! + (\theta i)^9/9! + (\theta i)^{10}/10! + (\theta i)^{11}/11! + \dots$

We know how to evaluate an imaginary number raised to an integer power, which is done as such:

$i^1 = i$

$i^2 = -1$ *terms repeat every four*

$i^3 = -i$

$i^4 = 1$

$i^5 = i$
$i^6 = -1$
etc...
We can see that it repeats every four terms. Knowing this, we can simplify the above expansion:
$e^{(\theta i)} = 1 + \theta i - \theta^2/2! - i\theta^3/3! + \theta^4/4! + i\theta^5/5! - \theta^6/6! - i\theta^7/7! + \theta^8/8! + i\theta^9/9! - \theta^{10}/10! - i\theta^{11}/11! + \ldots$
It just so happens that this power series can be broken up into two very convenient series:

$e^{(\theta i)} =$
$[1 - \theta^2/2! + \theta^4/4! - \theta^6/6! + \theta^8/8! - \theta^{10}/10! + \ldots]$
$+$
$[i\theta - i\theta^3/3! + i\theta^5/5! - i\theta^7/7! + i\theta^9/9! - i\theta^{11}/11! + \ldots]$
Now, look at the series expansions for sine and cosine. The above above equation happens to include those two series. The above equation can therefore be simplified to
$e^{(\theta i)} = \cos(\theta) + i \sin(\theta)$
An interesting case is when we set $\theta = \pi$, since the above equation becomes
$e^{(\pi i)} = -1 + 0i = -1.$
which can be rewritten as
$e^{(\pi i)} + 1 = 0$. *special case*
which remarkably links five very fundamental constants of mathematics into one small equation.

Again, this is not necessarily a proof since we have not shown that the sin(x), COs(x), and e^x series converge as indicated for imaginary numbers.]]

$n^{(a + BI)} = (\cos(b \ln n) + i \sin(b \ln n))n^a$

if $z = r(\cos \theta + i \sin \theta)$ then $z^n = r^n (\cos n\theta + i \sin n\theta)$(DeMoivre's Theorem)

if $w = r(\cos \theta + i \sin \theta)$;n=integer, then there are n complex nth roots (z) of w for k=0,1,..n-1:

$z(k) = r^{(1/n)} [\cos((\theta + 2(PI)k)/n) + i \sin((\theta + 2(PI)k)/n)]$

if $z = r (\cos \theta + i \sin \theta)$ then $\ln(z) = \ln r + i \theta$

$\sin(a + BI) = \sin(a)\cosh(b) + \cos(a)\sinh(b) i$

$\cos(a + BI) = \cos(a)\cosh(b) - \sin(a)\sinh(b) i$

$\tan(a + BI) = (\tan(a) + i \tanh(b)) / (1 - i \tan(a) \tanh(b))$
$= (\mathrm{sech}^2(b)\tan(a) + \sec^2(a)\tanh(b) i) / (1 + \tan^2(a)\tanh^2(b))$

8) Math | Miscellaneous | Complexity | $i^{\wedge 2}$)

Rationale that $i^2 = -1$

We know that by definition
$i = \sqrt{-1}$
Therefore,
$i^2 = [\sqrt{-1}]^2 = -1$.

9) Math | Miscellaneous | math table : Vectors

Prelude: A vector, as defined below, is a specific mathematical structure. It has numerous physical and geometric applications, which result mainly from its ability to represent **magnitude** and **direction** simultaneously. Wind, for example, had both a speed and a direction and, hence, is conveniently expressed as a vector. The same can be said of moving objects and forces. The location of a points on a cartesian coordinate plane is usually expressed as an ordered pair (x, y), which is a specific example of a vector. Being a vector, (x, y) has a a certain distance (magnitude) from and angle (direction) relative to the origin (0, 0). Vectors are quite useful in simplifying problems from three-dimensional geometry.

Definition:A **scalar**, generally speaking, is another name for "real number."

Definition: A **vector** of dimension n is an ordered collection of n elements, which are called **components**.

Notation: We often represent a vector by some letter, just as we use a letter to denote a scalar (real number) in algebra. In typewritten work, a vector is usually given a bold letter, such as **A**, to distinguish it from a scalar quantity, such as *A*. In handwritten work, writing bold letters is difficult, so we typically just place a right-handed arrow over the letter to denote a vector. An n-dimensional vector **A** has n elements denoted as A1, A2, ..., An. Symbolically, this can be written in multiple ways:

A = <A1, A2, ..., An>
A = (A1, A2, ..., An)

Example: (2,-5), (-1, 0, 2), (4.5), and (PI, a, b, 2/3) are all examples of vectors of dimension 2, 3, 1, and 4 respectively. The first vector has components 2 and -5.

Note: Alternately, an "unordered" collection of n elements {A1, A2, ..., An} is called a "set."

Definition: Two vectors are **equal** if their corresponding components are equal.

Example: If **A** = (-2, 1) and **B** = (-2, 1), then **A** = **B** since -2 = -2 and 1 = 1. However, (5, 3) not_equal (3, 5) because even though they have the same components, 3 and 5, the component do not occur in the same order. Contrast this with sets, where {5, 3} = {3, 5}.

Definition: The magnitude of a vector **A** of dimension n, denoted |**A**|, is defined as

|**A**| = sqrt(A1^2 + A2^2 + ... + An^2)

Geometrically speaking, magnitude is synonymous with "length," "distance", or "speed." In the two-dimensional case, the point represented by the vector A = (A1, A2) has a distance from the origin (0, 0) of sqrt(A1^2 + A2^2) according to the pythagorean theorem. In the three-dimension case, the point represented by the vector A = (A1, A2, A3) has a distance from the origin of sqrt(A1^2 + A2^2 + A3^2) according to the three-dimensional form of the Pythagorean theorem (A box with sides a, b, and c has a diagonal of length sqrt(a2+b2+c2)). With vectors of dimension n greater than three, our geometric intuition fails, but the algebraic definition remains.

Definition: The **sum** of two vectors **A** = (A1, A2, ..., An) and **B** = (B1, B2, ..., Bn) is defined as

A + **B** = (A1 + B1, A2 + B2, ..., An + Bn)

Note: Addition of vectors is only defined if both vectors have the same dimension.

Example:

(2, -3) + (0, 1) = (2+0, -3+1) = (2, -2).
(0.1, 2) + (-1, PI) = (0.1 + -1, 2 + PI) = (-0.9, 2+PI)

Justification: Physical and geometric applications warrant such a definition. IF a train travels East at 5 meters/second relative to the ground, which will be denoted in vector notation as VT = (0, 5), and a person on the train walks South at 1 meter/second relative to the train, which will be denoted as VP = (-1, 0), THEN the direction and speed that the person is traveling relative to the ground is represented by the vector VG = VT + VP = (0, 5) + (-1, 0) = (0 + -1, 5 + 0) = (-1, 5). This vector has a magnitude of |VG| = sqrt((-1)^2 + 5^2) = sqrt(6) = 2.449..., which means that the person is traveling at about 2.449 meters/second relative to the ground and the net direction is mostly East but slightly South.

Definition: The **scalar product** of a scalar k by a vector **A** = (A1, A2, ..., An) is defined as

k**A** = (kA1, kA2, ..., kAn)

Example:

2(5, -4) = (2*5, 2*-4) = (10, -8)
-3(1, 2) = (-3*1, -3*2) = (-3, -6)
0(3, 1) = (0*3, 0*1) = (0, 0)
1(2, 3) = (1*2, 1*3) = (2, 3)

Note: In general, 0**A** = (0, 0, ..., 0) and 1**A** = **A**, just as in the algebra of scalars. The vector of any dimension n with all zero elements (0, 0, ..., 0) is called the zero vector and is denoted **0**.

See also: Vector Definitions

Vector Notation: The lower case letters a-h, l-z denote scalars. Uppercase bold **A**-**Z** denote vectors. Lowercase bold **i**, **j**, **k** denote unit vectors. $\langle a, b\rangle$ denotes a vector with components a and b. $\langle x_1, .., x_n\rangle$ denotes vector with n components of which are $x_1, x_2, x_3, .., x_n$. $|\mathbf{R}|$ denotes the magnitude of the vector **R**.

$|\langle a, b\rangle|$ = magnitude of vector = $\sqrt{(a^2 + b^2)}$

$|\langle x_1, .., x_n\rangle| = \sqrt{(x_1^2 + .. + x_n^2)}$

$\langle a, b\rangle + \langle c, d\rangle = \langle a+c, b+d\rangle$

$\langle x_1, .., x_n\rangle + \langle y_1, .., y_n\rangle = \langle x_1+y_1, .., x_n+y_n\rangle$

$k \langle a, b\rangle = \langle ka, kb\rangle$

$k \langle x_1, .., x_n\rangle = \langle k x_1, .., k x_2\rangle$

$\langle a, b\rangle \cdot \langle c, d\rangle = ac + bd$

$\langle x_1, .., x_n\rangle \cdot \langle y_1, .., y_n\rangle = x_1 y_1 + .. + x_n y_n\rangle$

$\mathbf{R} \cdot \mathbf{S} = |\mathbf{R}|\ |\mathbf{S}| \cos\theta$ (θ = angle between them)

$\mathbf{R} \cdot \mathbf{S} = \mathbf{S} \cdot \mathbf{R}$

$(a\mathbf{R}) \cdot (b\mathbf{S}) = (ab)\, \mathbf{R} \cdot \mathbf{S}$

$\mathbf{R} \cdot (\mathbf{S} + \mathbf{T}) = \mathbf{R} \cdot \mathbf{S} + \mathbf{R} \cdot \mathbf{T}$

$\mathbf{R} \cdot \mathbf{R} = |\mathbf{R}|^2$

$|\mathbf{R} \times \mathbf{S}| = |\mathbf{R}|\ |\mathbf{S}| \sin\theta$ (θ = angle between both vectors). Direction of **R** x **S** is perpendicular to **A** & **B** and according to the right hand rule.

$$\mathbf{R} \times \mathbf{S} = \begin{vmatrix} i & j & k \\ r_1 & r_2 & r_3 \\ s_1 & s_2 & s_3 \end{vmatrix} = \left(\begin{vmatrix} r_2 & r_3 \\ s_2 & s_3 \end{vmatrix}, \begin{vmatrix} r_3 & r_1 \\ s_3 & s_1 \end{vmatrix}, \begin{vmatrix} r_1 & r_2 \\ s_1 & s_2 \end{vmatrix} \right)$$

$\mathbf{S} \times \mathbf{R} = -\mathbf{R} \times \mathbf{S}$

$(a\mathbf{R}) \times \mathbf{S} = \mathbf{R} \times (a\mathbf{S}) = a(\mathbf{R} \times \mathbf{S})$

$\mathbf{R} \times (\mathbf{S} + \mathbf{T}) = \mathbf{R} \times \mathbf{S} + \mathbf{R} \times \mathbf{T}$

R x **R** = 0

If a, b, c = angles between the unit vectors **i**, **j**,**k** and R Then the direction cosines are set by:

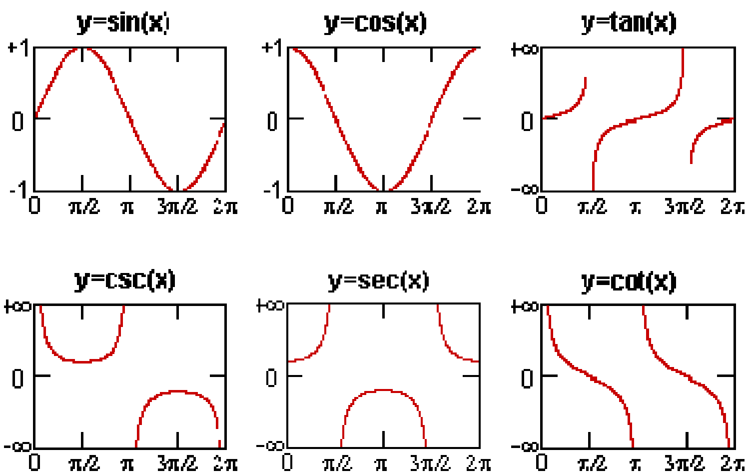
COs a = (**R** • **i**) / |**R**|; COs b = (**R** • **j**) / |**R**|; COs c = (**R** • **k**) / |**R**|

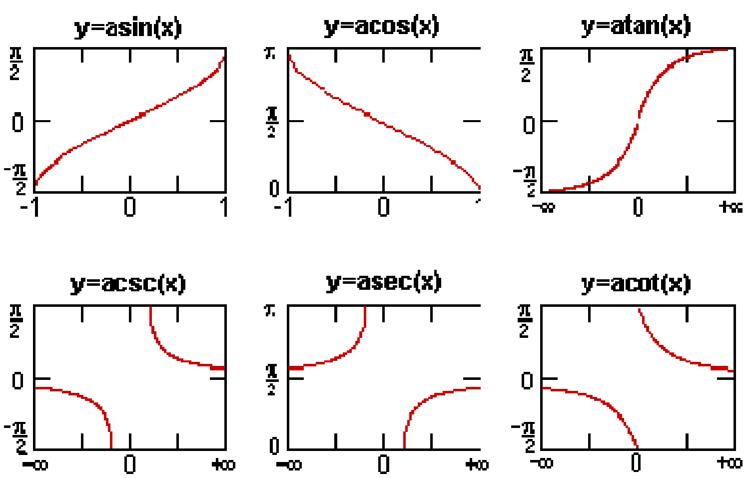
|**R** x **S**| = Area of parrallagram with sides **R**and **S**.

Component of **R** in the direction of **S** = |**R**|COs θ= (**R** • **S**) / |**S**|(scalar result)

Projection of **R** in the direction of **S** = |**R**|COs θ= (**R** • **S**) **S**/ |**S**| ^{2} (vector result)

10) Math | Trig | Trigonometric Graphs





11) Unprove theorem

Riemann Hypothesis

zeta(s) = $1/1^s + 1/2^s + 1/3^s + \ldots$ (s = a + it) all 0's of zeta(s) in strip 0<=a<=1 lie on central line a=1/2

Twin Primes occur infinitely

Twin primes are primes that are 2 integers apart. Examples include 5 & 7, 17 & 19, 101 & 103

Goldbach's Postulate

Every even # > 2 can be expressed as the sum of 2 primes.
4=2+2, 6=3+3, 8=3+5, 10=5+5, 12=5+7, .. , 100=3+97, ...

Euclid's Parallel Postulate

Through a point, not on a line, there exists exactly 1 line parallel to the given line. (Then there's those non-Euclidean people...)

$\sum_{(k=1..\infty)} 1/k^n = ?$

Although others have found that this expression equals $PI^2 / 6$ when n=2, $PI^4 / 90$ when n = 4 and similar solutions for all possible even values of n, no one has discovered an ***exact*** value when n is an odd integer (3, 5, 7, ...) (note: when n=1, the sum does not converge, but it does has relations to the gamma constant).

Trairashiq bargia sutra and rana's constant

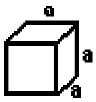
12) **Math | Geometry | Volume Formulas**

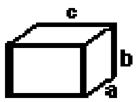
pi = π= 3.141592...)

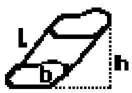
Volume Formulas

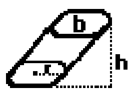
Note: "ab" means "a" multiplied by "b". "a^2" means "a squared", which is the same as "a" times "a". "b^3" means "b cubed", which is the same as "b" times "b" times "b".

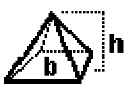
Be careful!! Units count. Use the same units for all measurements. Examples

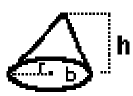
cube = a^3 

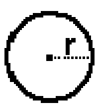
rectangular prism = a b c 

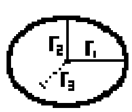
irregular prism = **b** h 

cylinder = **b** h = *pi* r^2 h 

pyramid = (1/3) **b** h 

cone = (1/3) **b** h = 1/3 *pi* r^2 h 

sphere = (4/3) *pi* r^3 

ellipsoid = (4/3) *pi* r_1 r_2 r_3 

Units

Volume is measured in "cubic" units. The volume of a figure is the number of cubes required to fill it completely, like blocks in a box.

Volume of a cube = side times side times side. Since each side of a square is the same, it can simply be the length of one side cubed.

If a square has one side of 4 inches, the volume would be 4 inches times 4 inches times 4 inches, or 64

cubic inches. (Cubic inches can also be written in^3.)

Be sure to use the same units for all measurements. You cannot multiply feet times inches times yards, it doesn't make a perfectly cubed measurement.

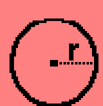
The volume of a rectangular prism is the length on the side times the width times the height. If the width is 4 inches, the length is 1 foot and the height is 3 feet, what is the volume?

NOT CORRECT .... 4 times 1 times 3 = 12

CORRECT.... 4 inches is the same as 1/3 feet. Volume is 1/3 feet times 1 foot times 3 feet = 1 cubic foot (or 1 cu. ft., or 1 ft^3).

Geometry

13) Circles :



a circle

Definition: A circle is the locus of all points equidistant from a central point.

Definitions Related to Circles

arc: a curved line that is part of the circumference of a circle

chord: a line segment within a circle that touches 2 points on the circle.

circumference: the distance around the circle.

diameter: the longest distance from one end of a circle to the other.

origin: the center of the circle

pi (π): A number, 3.141592..., equal to (the circumference) / (the diameter) of any circle.

radius: distance from center of circle to any point on it.

sector: is like a slice of pie (a circle wedge).

tangent of circle: a line perpendicular to the radius that touches ONLY one point on the circle.

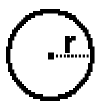
Diameter = 2 x radius of circle

Circumference of Circle = **PI x diameter** = 2 PI x radius

where **PI = π = 3.141592...**

Area of Circle:

area = PI r^2



Length of a Circular Arc: (with central angle θ)

if the angle θ is in degrees, then length = θ x (PI/180) x r

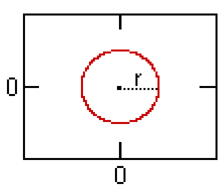
if the angle θ is in radians, then length = r x θ

Area of Circle Sector: (with central angle θ)

if the angle θ is in degrees, then area = (θ/360) x PI r^2

if the angle θ is in radians, then area = ((θ/(2PI)) x PI r^2

Equation of Circle: (Cartesian coordinates)



for a circle with center **(j, k)** and radius **(r):**

(x-j)^2 + (y-k)^2 = r^2

Equation of Circle: (polar coordinates)

for a circle with center (0, 0): **r(θ) = radius**

for a circle with center with polar coordinates: (c, α) and radius **a**:

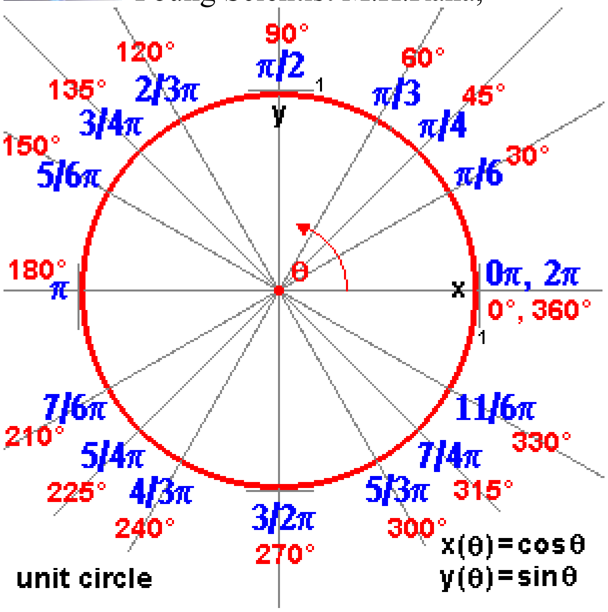
$$r^2 - 2cr\cos(\theta - \alpha) + c^2 = a^2$$

Equation of a Circle: (parametric coordinates)

for a circle with origin (j, k) and radius r:

x(t) = r cos(t) + j y(t) = r sin(t) + k

Young Scientist M.H.Rana, www.matherana.synthasite.com



14) **Polygon Properties**

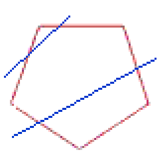
What is a Polygon?

A closed plane figure made up of several line segments that are joined together. The sides do not cross each other. Exactly two sides meet at every vertex.

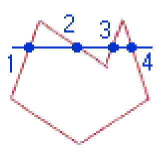
Types | Formulas | Parts | Special Polygons | Names

Types of Polygons

Regular - all angles are equal and all sides are the same length. Regular polygons are both equiangular and equilateral.
Equiangular - all angles are equal.
Equilateral - all sides are the same length.



Convex - a straight line drawn through a convex polygon **crosses at most two sides**. Every interior angle is less than 180°.



Concave - you can draw at least one straight line through a concave polygon that **crosses more than two sides**. At least one interior angle is more than 180°.

Polygon Formulas

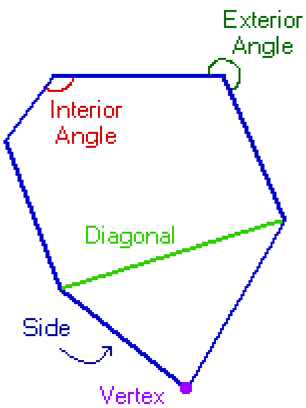
(N = # of sides and S = length from center to a corner)

Area of a regular polygon = (1/2) N sin(360°/N) S^2

Sum of the interior angles of a polygon = (N - 2) x 180°

The **number of diagonals** in a polygon = 1/2 N(N-3)
The **number of triangles** (when you draw all the diagonals from one vertex) in a polygon = (N - 2)

Polygon Parts



Side - one of the line segments that make up the polygon.

Vertex - point where two sides meet. Two or more of these points are called vertices.

Diagonal - a line connecting two vertices that isn't a side.

Interior Angle - Angle formed by two adjacent sides inside the polygon.

Exterior Angle - Angle formed by two adjacent sides outside the polygon.

Special Polygons

Special Quadrilaterals - square, rhombus, parallelogram, rectangle, and the trapezoid.

Special Triangles - right, equilateral, isosceles, scalene, acute, obtuse.

Polygon Names

Generally accepted names

Sides	Name
n	N-gon
3	Triangle
4	Quadrilateral
5	Pentagon
6	Hexagon
7	Heptagon
8	Octagon
10	Decagon
12	Dodecagon

 Young Scientist M.H.Rana, www.matherana.synthasite.com

Names for other polygons have been proposed.

Sides	Name
9	Nonagon, Enneagon
11	Undecagon, Hendecagon
13	Tridecagon, Triskaidecagon
14	Tetradecagon, Tetrakaidecagon
15	Pentadecagon, Pentakaidecagon
16	Hexadecagon, Hexakaidecagon
17	Heptadecagon, Heptakaidecagon
18	Octadecagon, Octakaidecagon
19	Enneadecagon, Enneakaidecagon
20	Icosagon
30	Triacontagon
40	Tetracontagon
50	Pentacontagon
60	Hexacontagon
70	Heptacontagon
80	Octacontagon
90	Enneacontagon
100	Hectogon, Hecatontagon
1,000	Chiliagon
10,000	Myriagon

To construct a name, combine the prefix+suffix

Sides	Prefix		Sides	Suffix
20	Icosikai...		+1	...henagon
30	Triacontakai...		+2	...digon
40	Tetracontakai...		+3	...trigon
50	Pentacontakai...		+4	...tetragon
60	Hexacontakai...	+	+5	...pentagon
70	Heptacontakai...		+6	...hexagon
80	Octacontakai...		+7	...heptagon
90	Enneacontakai...		+8	...octagon
			+9	...enneagon

Examples:
46 sided polygon - Tetracontakaihexagon

28 sided polygon - Icosikaioctagon

However, many people use the form n-gon, as in 46-gon, or 28-gon instead of these names.

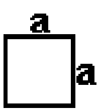
15) **Area Formulas**

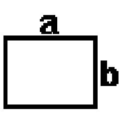
pi **= π = 3.141592...)**

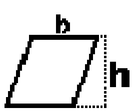
Area Formulas

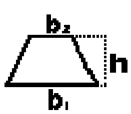
Note: "ab" means "a" multiplied by "b". "a^2" means "a squared", which is the same as "a" times "a".

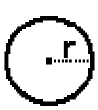
Be careful!! Units count. Use the same units for all measurements. Examples

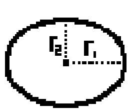
square = a^2 

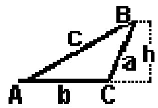
rectangle = ab 

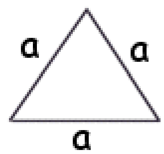
parallelogram = bh 

trapezoid = $h/2\ (b_1 + b_2)$ 

circle = $pi\ r^2$ 

ellipse = $pi\ r_1\ r_2$ 

triangle =	$\frac{1}{2}(bh)$		one half times the base length times the height of the triangle

equilateral triangle =	$\frac{\sqrt{3}}{4}(a^2)$ 

triangle given SAS (two sides and the opposite angle)
= (1/2) a b sin C

triangle given a,b,c = √[s(s-a)(s-b)(s-c)] when s = (a+b+c)/2 **(Heron's formula)**

regular polygon = (1/2) n sin(360°/n) S^2
when n = # of sides and S = length from center to a corner

Units

Area is measured in "square" units. The area of a figure is the number of squares required to cover it completely, like tiles on a floor.

Area of a square = side times side. Since each side of a square is the same, it can simply be the length of one side squared.

If a square has one side of 4 inches, the area would be 4 inches times 4 inches, or 16 square inches. (Square inches can also be written in^2.)

Be sure to use the same units for all measurements. You cannot multiply feet times inches, it doesn't make a square measurement.

The area of a rectangle is the length on the side times the width. If the width is 4 inches and the length is 6 feet, what is the area?

NOT CORRECT .... 4 times 6 = 24

CORRECT.... 4 inches is the same as 1/3 feet. Area is 1/3 feet times 6 feet = 2 square feet. (or 2 sq. ft., or 2 ft^2).

16) Perimeter

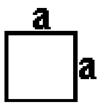
pi **= π= 3.141592...)**

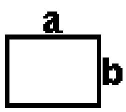
Perimeter Formulas

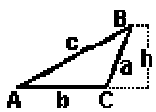
The perimeter of any polygon is the sum of the lengths of all the sides.

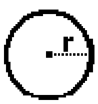
Note: "ab" means "a" multiplied by "b". "a^2" means "a squared", which is the same as "a" times "a".

Be careful!! Units count. Use the same units for all measurements. Examples

square = 4a 

rectangle = 2a + 2b 

triangle = a + b + c 

circle = *2pi* r 

circle = *pi* d (where d is the diameter)

The perimeter of a circle is more commonly known as the circumference.

Units

Be sure to only add similar units. For example, you cannot add inches to feet.

For example, if you need to find the perimeter of a rectangle with sides of 9 inches and 1 foot, you must first change to the same units.

perimeter = 2 (a + b)

INCORRECT
perimeter = 2(9 + 1) = 2*10 = 20

CORRECT
perimeter = 2(9 inches + 1 foot)
= 2(3/4 foot + 1 foot)
= 2(1 3/4 feet)
= 3 1/2 feet

17) **Surface Area Formulas)**

(*pi* = π = 3.141592...)

Surface Area Formulas

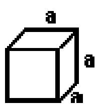
In general, the surface area is the sum of all the areas of all the shapes that cover the surface of the object.

Cube | Rectangular Prism | Prism | Sphere | Cylinder | Units

Note: "ab" means "a" multiplied by "b". "a^2" means "a squared", which is the same as "a" times "a".

Be careful!! Units count. Use the same units for all measurements. Examples

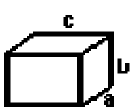
Surface Area of a Cube = 6 a^2



(a is the length of the side of each edge of the cube)

In words, the surface area of a cube is the area of the six squares that cover it. The area of one of them is a*a, or a^2. Since these are all the same, you can multiply one of them by six, so the surface area of a cube is 6 times one of the sides squared.

Surface Area of a Rectangular Prism = 2ab + 2bc + 2ac

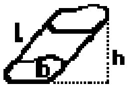


(a, b, and c are the lengths of the 3 sides)

In words, the surface area of a rectangular prism is the area of the six rectangles that cover it. But we don't have to figure out all six because we know that the top and bottom are the same, the front and back are the same, and the left and right sides are the same.

The area of the top and bottom (side lengths a and c) = a*c. Since there are two of them, you get 2ac. The front and back have side lengths of b and c. The area of one of them is b*c, and there are two of them, so the surface area of those two is 2bc. The left and right side have side lengths of a and b, so the surface area of one of them is a*b. Again, there are two of them, so their combined surface area is 2ab.

Surface Area of Any Prism



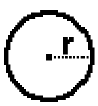
(b is the shape of the ends)

Surface Area = Lateral area + Area of two ends

(Lateral area) = (perimeter of shape **b**) * L

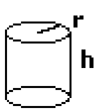
Surface Area = (perimeter of shape **b**) * L+ 2*(Area of shape **b**)

Surface Area of a Sphere = 4 *pi* r ^{2}



(r is radius of circle)

Surface Area of a Cylinder = 2 *pi* r ^{2} + 2 *pi* r h



(h is the height of the cylinder, r is the radius of the top)

Surface Area = Areas of top and bottom +Area of the side

Surface Area = 2(Area of top) + (perimeter of top)* height

Surface Area = 2(*pi* r ^{2}) + (2 *pi* r)* h

In words, the easiest way is to think of a can. The surface area is the areas of all the parts needed to cover the can. That's the top, the bottom, and the paper label that wraps around the middle.

You can find the area of the top (or the bottom). That's the formula for area of a circle (*pi* r ^{2}). Since there is both a top and a bottom, that gets multiplied by two.

The side is like the label of the can. If you peel it off and lay it flat it will be a rectangle. The area of a rectangle is the product of the two sides. One side is the height of the can, the other side is the perimeter of the circle, since the label wraps once around the can. So the area of the rectangle is (2 *pi* r)* h.

Add those two parts together and you have the formula for the surface area of a cylinder.

Surface Area = 2(*pi* r ^{2}) + (2 *pi* r)* h

Tip! Don't forget the units.

These equations will give you correct answers if you keep the units straight. For example - to find the surface area of a cube with sides of 5 inches, the equation is:

Surface Area = 6*(5 inches)^2

= 6*(25 square inches)

= 150 sq. inches

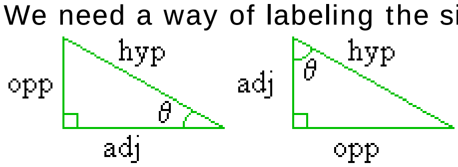
Trigonometry

18)**Math** | **Trig** |**LableSides**)

Trig: Labeling Sides

Since there are three sides and two non-right angles in a right triangle, the trigonometric functions will need a way of specifying which sides are related to which angle. (It is not-so-useful to know that the ratio of the lengths of two sides equals 2 if we do not know which of the three sides we are talking about. Likewise, if we determine that one of the angles is 40°, it would be nice to know of which angle this statement is true.

We need a way of labeling the sides. Consider a general right triangle:



A right triangle has two non-right angles, and we choose one of these angles to be our angle of interest, which we label "q." ("q" is the Greek letter "theta.")

We can then uniquely label the three sides of the right triangle *relative to* our choice of q. As the above picture illustrates, our choice of q affects how the three sides get labeled.

We label the three sides in this manner: The side opposite the right angle is called **the hypotenuse**. This side is labeled the same regardless of our choice of q. The labeling of the remaining two sides depend on our choice of theta; we therefore speak of these other two sides as being **adjacent to** the angle q or **opposite to** the angle q. The remaining side that touches the angle q is considered to be the side **adjacent to** q, and the remaining side that is far away from the angle q is considered to be **opposite to** the angle q, as shown in the picture.

19) **Math** | **Algebra** | **Function** |**Overview**

Trig Functions: Overview

Under its simplest definition, a trigonometric (literally, a "triangle-measuring") function, is one of the many functions that relate one non-right angle of a **right triangle** to the ratio of the lengths of any two sides of the triangle (or vice versa).

Any trigonometric function (f), therefore, always satisfies either of the following equations:

f(q) = a / b *OR* f(a / b) = q,
where q is the measure of a certain angle in the triangle, and a and b are the lengths of two specific sides.

This means that

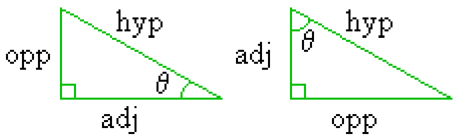
- If the former equation holds, we can choose any right triangle, then take the measurement of one of the non-right angles, and when we evaluate the trigonometric function at that *angle*, the result will be the *ratio of the lengths of two of the triangle's sides*.
- However, if the latter equation holds, we can chose any right triangle, then compute the ratio of the lengths of two specific sides, and when we evaluate the trigonometric function at any that *ratio*, the result will be measure of one of the triangles non-right angles. (These are called **inverse trig functions** since they do the inverse, or vice-versa, of the previous trig functions.)

This relationship between an angle and side ratios in a right triangle is one of the most important ideas in trigonometry. Furthermore, trigonometric functions work for *any* right triangle. Hence -- for a right triangle -- if we take the measurement of one of the triangles non-right angles, we can mathematically deduce the ratio of the lengths of any two of the triangle's sides by trig functions. And if we measure any side ratio, we can mathematically deduce the measure of one of the triangle's non-right angles by inverse trig functions. More importantly, if we know the measurement of one of the triangle's angles, and we then use a trigonometric function to determine the ratio of the lengths of two of the triangle's sides, *and* we happen to know the lengths of one of these sides in the ratio, we can then algebraically determine the length of the other one of these two sides. (i.e. if we determine that a / b = 2, and we know a = 6, then we deduce that b = 3.)

Since there are three sides and two non-right angles in a right triangle, the trigonometric functions will need a way of specifying which sides are related to which angle. (It is not-so-useful to know that

the ratio of the lengths of two sides equals 2 if we do not know which of the three sides we are talking about. Likewise, if we determine that one of the angles is 40°, it would be nice to know of which angle this statement is true.

Under a certain convention, we label the sides as **opposite**, **adjacent**, and **hypotenuse** *relative* to our angle of interest q. full explanation



As mentioned previously, the first type of trigonometric function, which relates an angle to a side ratio, always satisfies the following equation:

f(q) = a / b.
Since given any angle q, there are three ways of choosing the numerator (a), and three ways of choosing the denominator (b), we can create the following nine trigonometric functions:

f(q) = opp/opp	f(q) = opp/adj	f(q) = opp/hyp
f(q) = adj/opp	f(q) = adj/adj	f(q) = adj/hyp
f(q) = hyp/opp	f(q) = hyp/adj	f(q) = hyp/hyp

The three diagonal functions shown in red always equal one. They are degenerate and, therefore, are of no use to us. We therefore remove these degenerate functions and assign labels to the remaining six, usually written in the following order:

sine(q) = opp/hyp	cosecant(q) = hyp/opp
cosine(q) = adj/hyp	secant(q) = hyp/adj
tangent(q) = opp/adj	cotangent(q) = adj/opp

Furthermore, the functions are usually abbreviated: sine (sin), cosine (cos), tangent (tan) cosecant (csc), secant (sec), and cotangent (cot).

Do not be overwhelmed. By far, the two most important trig functions to remember are sine and cosine. All the other trig functions of the first kind can be derived from these two functions. For example, the functions on the right are merely the multiplicative inverse of the corresponding function on the left (that makes them much less useful). Furthermore, the sin(x) / COs(x) = (opp/hyp) / (adj/hyp) = opp / adj = tan(x). Therefore, the tangent function is the same as the quotient of the sine and cosine functions (the tangent function is still fairly handy).

sine(q) = opp/hyp	CSC(q) = 1/sin(q)
COs(q) = adj/hyp	sec(q) = 1/COs(q)
tan(q) = sin(q)/COs(q)	cot(q) = 1/tan(q)

Let's examine these functions further. You will notice that there are the sine, secant, and tangent functions, and there are corresponding "co"-functions. They get their odd names from various similar ideas in geometry. You may suggest that the cofunctions *should* be relabeled to be the multiplicative inverses of the corresponding sine, secant, and tangent functions. However, there is a method to this madness. A **cofunction** of a given trig function (f) is, by definition, the function obtained after the **complement** its parameter is taken. Since the complement of any angle q is 90° - q, the the fact that the following relations can be shown to hold:

sine(90° - q) = cosine(q)

secant(90° - q) = cosecant(q)
tangent(90° - q) = cotangent(q)
thus justifying the naming convention.

The trig functions evaluate differently depending on the units on q, such as **degrees, radians, or grads**. For example, sin(90°) = 1, while sin(90)=0.89399.... explanation

Just as we can define trigonometric functions of the form

f(q) = a / b
that take a non-right angle as its parameter and return the ratio of the lengths of two triangle sides, we can do the reverse: define trig functions of the form
f(a / b) = q
that take the ratio of the lengths of two sides as a parameter and returns the measurement of one of the non-right angles.

Inverse Functions	
arcsine(opp/hyp) = q	arccosecant(hyp/opp) = q
arccosine(adj/hyp) = q	arcsecant(hyp/adj) = q
arctangent(opp/adj) = q	arccotangent(adj/opp) = q

As before, the functions are usually abbreviated: arcsine (arcsin), arccosine (arccos), arctangent (arctan) arccosecant (arccsc), arcsecant (arcsec), and arccotangent (arccot). According to the standard notation for inverse functions (f^{-1}), you will also often see these written as $\sin^{-1}$, cos-1, $\tan^{-1}$ csc-1, $\sec^{-1}$, and $\cot^{-1}$. *Beware*: There is another common notation that writes the square of the trig functions, such as $(\sin(x))^2$ as $\sin^2(x)$. This can be confusing, for you then *might* then be lead to think that $\sin^{-1}(x) = (\sin(x))^{-1}$, which is *not* true. The negative one superscript here is a special notation that denotes inverse functions (not multiplicative inverses).

20) **Math | Algebra | Trig | Unit Modes**

Trig Functions: Unit Modes

The trig functions evaluate differently depending on the units on q. For example, sin(90°) = 1, while sin(90)=0.89399.... If there is a degree sign after the angle, the trig function evaluates its parameter as a degree measurement. If there is no unit after the angle, the trig function evaluates its

parameter as a radian measurement. This is because radian measurements are considered to be the "natural" measurements for angles. (Calculus gives us a justification for this. A partial explanation comes from the formula for the area of a circle sector, which is simplest when the angle is in radians).

Calculator note: Many calculators have **degree**, **radian**, and **grad** modes (360° = 2p rad = 400 grad). It is important to have the calculator in the right mode since that mode setting tells the calculator which units to assume for angles when evaluating any of the trigonometric functions. For example, if the calculator is in degree mode, evaluating sine of 90 results in 1. However, the calculator returns 0.89399... when in radian mode. Having the calculator in the wrong mode is a common mistake for beginners, especially those that are only familiar with degree angle measurements.

For those who wish to reconcile the various trig functions that depend on the units used, we can *define* the degree symbol (°) to be the value (PI/180). Therefore, sin(90°), for example, is really just an expression for the sine of a radian measurement when the parameter is fully evaluated. As a demonstration, sin(90°) = sin(90(PI/180)) = sin(PI/2). In this way, we only need to tabulate the "natural" radian version of the sine function. (This method is similar to defining percent % = (1/100) in order to relate percents to ratios, such as 50% = 50(1/100) = 1/2.)

20)

Trigonometric Tables

(Math | Trig | Tables)

PI = 3.141592... (approximately 22/7 = 3.1428)
radians = degrees x PI / 180 **(deg to rad conversion)**
degrees = radians x 180 / PI **(rad to deg conversion)**

Rad	Deg	Sin	Cos	Tan	Csc	Sec	Cot		
.0000	**00**	.0000	1.0000	.0000	-----	1.0000	-----	**90**	**1.5707**
.0175	**01**	.0175	.9998	.0175	57.2987	1.0002	57.2900	**89**	**1.5533**
.0349	**02**	.0349	.9994	.0349	28.6537	1.0006	28.6363	**88**	**1.5359**
.0524	**03**	.0523	.9986	.0524	19.1073	1.0014	19.0811	**87**	**1.5184**
.0698	**04**	.0698	.9976	.0699	14.3356	1.0024	14.3007	**86**	**1.5010**
.0873	**05**	.0872	.9962	.0875	11.4737	1.0038	11.4301	**85**	**1.4835**
.1047	**06**	.1045	.9945	.1051	9.5668	1.0055	9.5144	**84**	**1.4661**
.1222	**07**	.1219	.9925	.1228	8.2055	1.0075	8.1443	**83**	**1.4486**
.1396	**08**	.1392	.9903	.1405	7.1853	1.0098	7.1154	**82**	**1.4312**
.1571	**09**	.1564	.9877	.1584	6.3925	1.0125	6.3138	**81**	**1.4137**

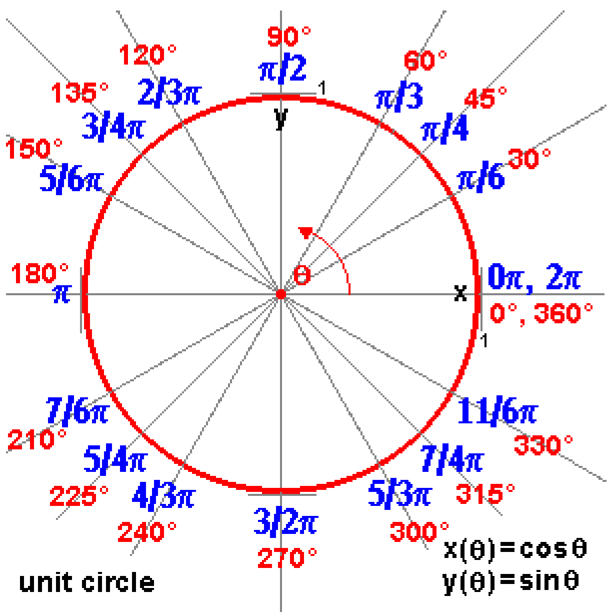
.1745	**10**	.1736	.9848	.1763	5.7588	1.0154	5.6713	**80**	**1.3953**
.1920	**11**	.1908	.9816	.1944	5.2408	1.0187	5.1446	**79**	**1.3788**
.2094	**12**	.2079	.9781	.2126	4.8097	1.0223	4.7046	**78**	**1.3614**
.2269	**13**	.2250	.9744	.2309	4.4454	1.0263	4.3315	**77**	**1.3439**
.2443	**14**	.2419	.9703	.2493	4.1336	1.0306	4.0108	**76**	**1.3265**
.2618	**15**	.2588	.9659	.2679	3.8637	1.0353	3.7321	**75**	**1.3090**
.2793	**16**	.2756	.9613	.2867	3.6280	1.0403	3.4874	**74**	**1.2915**
.2967	**17**	.2924	.9563	.3057	3.4203	1.0457	3.2709	**73**	**1.2741**
.3142	**18**	.3090	.9511	.3249	3.2361	1.0515	3.0777	**72**	**1.2566**
.3316	**19**	.3256	.9455	.3443	3.0716	1.0576	2.9042	**71**	**1.2392**
.3491	**20**	.3420	.9397	.3640	2.9238	1.0642	2.7475	**70**	**1.2217**
.3665	**21**	.3584	.9336	.3839	2.7904	1.0711	2.6051	**69**	**1.2043**
.3840	**22**	.3746	.9272	.4040	2.6695	1.0785	2.4751	**68**	**1.1868**
.4014	**23**	.3907	.9205	.4245	2.5593	1.0864	2.3559	**67**	**1.1694**
.4189	**24**	.4067	.9135	.4452	2.4586	1.0946	2.2460	**66**	**1.1519**
.4363	**25**	.4226	.9063	.4663	2.3662	1.1034	2.1445	**65**	**1.1345**
.4538	**26**	.4384	.8988	.4877	2.2812	1.1126	2.0503	**64**	**1.1170**
.4712	**27**	.4540	.8910	.5095	2.2027	1.1223	1.9626	**63**	**1.0996**
.4887	**28**	.4695	.8829	.5317	2.1301	1.1326	1.8807	**62**	**1.0821**
.5061	**29**	.4848	.8746	.5543	2.0627	1.1434	1.8040	**61**	**1.0647**
.5236	**30**	.5000	.8660	.5774	2.0000	1.1547	1.7321	**60**	**1.0472**
.5411	**31**	.5150	.8572	.6009	1.9416	1.1666	1.6643	**59**	**1.0297**
.5585	**32**	.5299	.8480	.6249	1.8871	1.1792	1.6003	**58**	**1.0123**
.5760	**33**	.5446	.8387	.6494	1.8361	1.1924	1.5399	**57**	**.9948**
.5934	**34**	.5592	.8290	.6745	1.7883	1.2062	1.4826	**56**	**.9774**
.6109	**35**	.5736	.8192	.7002	1.7434	1.2208	1.4281	**55**	**.9599**
.6283	**36**	.5878	.8090	.7265	1.7013	1.2361	1.3764	**54**	**.9425**
.6458	**37**	.6018	.7986	.7536	1.6616	1.2521	1.3270	**53**	**.9250**
.6632	**38**	.6157	.7880	.7813	1.6243	1.2690	1.2799	**52**	**.9076**
.6807	**39**	.6293	.7771	.8098	1.5890	1.2868	1.2349	**51**	**.8901**
.6981	**40**	.6428	.7660	.8391	1.5557	1.3054	1.1918	**50**	**.8727**
.7156	**41**	.6561	.7547	.8693	1.5243	1.3250	1.1504	**49**	**.8552**
.7330	**42**	.6691	.7431	.9004	1.4945	1.3456	1.1106	**48**	**.8378**
.7505	**43**	.6820	.7314	.9325	1.4663	1.3673	1.0724	**47**	**.8203**
.7679	**44**	.6947	.7193	.9657	1.4396	1.3902	1.0355	**46**	**.8029**
.7854	**45**	.7071	.7071	1.0000	1.4142	1.4142	1.0000	**45**	**.7854**

		COs	Sin	Cot	Sec	CSC	Tan	Deg	Rad

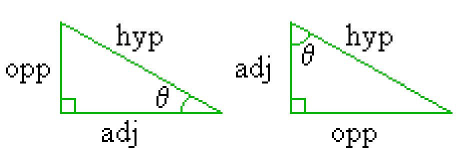
Trig Table of Common Angles

angle (degrees)	0	30	45	60	90	120	135	150	180	210	225	240	270	300	315	330	360 = 0
angle (radians)	0	PI/6	PI/4	PI/3	PI/2	2/3PI	3/4PI	5/6PI	PI	7/6PI	5/4PI	4/3PI	3/2PI	5/3PI	7/4PI	11/6PI	2PI = 0
sin(a)	√(0/4)	√(1/4)	√(2/4)	√(3/4)	√(4/4)	√(3/4)	√(2/4)	√(1/4)	√(0/4)	-√(1/4)	-√(2/4)	-√(3/4)	-√(4/4)	-√(3/4)	-√(2/4)	-√(1/4)	√(0/4)
COs(a)	√(4/4)	√(3/4)	√(2/4)	√(1/4)	√(0/4)	-√(1/4)	-√(2/4)	-√(3/4)	-√(4/4)	-√(3/4)	-√(2/4)	-√(1/4)	√(0/4)	√(1/4)	√(2/4)	√(3/4)	√(4/4)
tan(a)	√(0/4)	√(1/3)	√(2/2)	√(3/1)	√(4/0)	-√(3/1)	-√(2/2)	-√(1/3)	-√(0/4)	√(1/3)	√(2/2)	√(3/1)	√(4/0)	-√(3/1)	-√(2/2)	-√(1/3)	√(0/4)

Those with a zero in the denominator are undefined. They are included solely to demonstrate the pattern.



21) **Trig Functions: The Functions**



sine(q) = opp/hyp	cosecant(q) = hyp/opp
cosine(q) = adj/hyp	secant(q) = hyp/adj
tangent(q) = opp/adj	cotangent(q) = adj/opp

The functions are usually abbreviated: sine (sin), cosine (cos), tangent (tan) cosecant (csc), secant (sec), and cotangent (cot).

It is often simpler to memorize the the trig functions in terms of only sine and cosine:

sine(q) = opp/hyp	csc(q) = 1/sin(q)
cos(q) = adj/hyp	sec(q) = 1/COs(q)
tan(q) = sin(q)/cos(q)	cot(q) = 1/tan(q)

Inverse Functions	
arcsine(opp/hyp) = q	arccosecant(hyp/opp) = q
arccosine(adj/hyp) = q	arcsecant(hyp/adj) = q
arctangent(opp/adj) = q	arccotangent(adj/opp) = q

The functions are usually abbreviated:

arcsine (arcsin)
arccosine (arccos)
arctangent (arctan)
arccosecant (arccsc)
arcsecant (arcsec)
arccotangent (arccot).

According to the standard notation for inverse functions (f^{-1}), you will also often see these written as $\sin^{-1}$, cos-1, $\tan^{-1}$ arccsc^{-1}, arcsec^{-1}, and arccot^{-1}. ***Beware***, though, there is another common notation that writes the square of the trig functions, such as $(\sin(x))^2$ as $\sin^2(x)$. This can be confusing, for you then *might* then be lead to think that $\sin^{-1}(x) = (\sin(x))^{-1}$, which is *not* true. The negative one superscript here is a special notation that denotes inverse functions (not multiplicative inverses).

See also: overview.

22)

Proof: Hyperbolic Trigonometric Identities

(Math | Trig | Hyperbolas

Hyperbolic Definitions

$\sinh(x) = (e^{x} - e^{-x})/2$

$\operatorname{csch}(x) = 1/\sinh(x) = 2/(e^{x} - e^{-x})$

$\cosh(x) = (e^{x} + e^{-x})/2$

$\operatorname{sech}(x) = 1/\cosh(x) = 2/(e^{x} + e^{-x})$

$\tanh(x) = \sinh(x)/\cosh(x) = (e^{x} - e^{-x})/(e^{x} + e^{-x})$

$\coth(x) = 1/\tanh(x) = (e^{x} + e^{-x})/(e^{x} - e^{-x})$

$\cosh^{2}(x) - \sinh^{2}(x) = 1$

$\tanh^{2}(x) + \operatorname{sech}^{2}(x) = 1$

$\coth^{2}(x) - \operatorname{csch}^{2}(x) = 1$

Inverse Hyperbolic Definitions

$\operatorname{arcsinh}(z) = \ln(z + \sqrt{(z^{2} + 1)})$

$\operatorname{arccosh}(z) = \ln(z \pm \sqrt{(z^{2} - 1)})$

$\operatorname{arctanh}(z) = 1/2 \ln((1+z)/(1-z))$

$\operatorname{arccsch}(z) = \ln((1+\sqrt{(1+z^{2})})/z)$

$\operatorname{arcsech}(z) = \ln((1\pm\sqrt{(1-z^{2})})/z)$

$\operatorname{arccoth}(z) = 1/2 \ln((z+1)/(z-1))$

Relations to Trigonometric Functions

$\sinh(z) = -i \sin(iz)$

$\operatorname{csch}(z) = i \csc(iz)$

$\cosh(z) = \cos(iz)$

sech(z) = sec(iz)

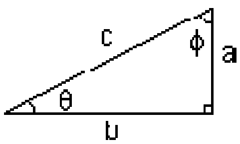
tanh(z) = -i tan(iz)

coth(z) = i cot(iz)

23)

Trigonometric Identities

(**Math** | **Trig** | **Identities**)



sin(theta) = a / c	csc(theta) = 1 / sin(theta) = c / a
cos(theta) = b / c	sec(theta) = 1 / cos(theta) = c / b
tan(theta) = sin(theta) / cos(theta) = a / b	cot(theta) = 1/ tan(theta) = b / a

sin(-x) = -sin(x)
csc(-x) = -csc(x)
cos(-x) = cos(x)
sec(-x) = sec(x)
tan(-x) = -tan(x)
cot(-x) = -cot(x)

$\sin^{2}(x) + \cos^{2}(x) = 1$	$\tan^{2}(x) + 1 = \sec^{2}(x)$	$\cot^{2}(x) + 1 = \csc^{2}(x)$
sin(x ± y) = sin x cos y ± cos x sin y		
cos(x ± y) = cos x cosy ∓ sin x sin y		

tan(x ± y) = (tan x ± tan y) / (1 ∓ tan x tan y)

sin(2x) = 2 sin x cos x

$\cos(2x) = \cos^{2}(x) - \sin^{2}(x) = 2\cos^{2}(x) - 1 = 1 - 2\sin^{2}(x)$

$\tan(2x) = 2\tan(x) / (1 - \tan^{2}(x))$

$\sin^{2}(x) = 1/2 - 1/2\cos(2x)$

$\cos^2(x) = 1/2 + 1/2 \cos(2x)$

$\sin x - \sin y = 2 \sin((x - y)/2) \cos((x + y)/2)$

$\cos x - \cos y = -2 \sin((x - y)/2) \sin((x + y)/2)$

Trig Table of Common Angles					
angle	**0**	**30**	**45**	**60**	**90**
$\sin^2$(a)	0/4	1/4	2/4	3/4	4/4
$\cos^2$(a)	4/4	3/4	2/4	1/4	0/4
$\tan^2$(a)	0/4	1/3	2/2	3/1	**4/0**

Given Triangle abc, with angles A,B,C; a is opposite to A, b opposite B, c opposite C:

$a/\sin(A) = b/\sin(B) = c/\sin(C)$ **(Law of Sines)**

$c^2 = a^2 + b^2 - 2ab \cos(C)$

$b^2 = a^2 + c^2 - 2ac \cos(B)$ **(Law of Cosines)**

$a^2 = b^2 + c^2 - 2bc \cos(A)$

$(a - b)/(a + b) = \tan [(A-B)/2] / \tan [(A+B)/2]$ **(Law of Tangents)**

Calculus

24)

Proof: *Constant Rule*

(**Math** | **Calculus** | **Derivatives** | **Identities** | **Constant Rule**

$\frac{d}{dx} c\, f(x) = c \frac{d}{dx} f(x)$

Proof of $\frac{d}{dx} c\, f(x) = c \frac{d}{dx} f(x)$ from the definition

We can use the definition of the derivative:

$$\frac{d}{dx}f(x) = \lim_{d \to 0} \frac{f(x+d)-f(x)}{d}$$

Therefore, $\frac{d}{dx}c\ f(x)$ can be written as such:

$$\frac{d}{dx}\mathbf{c\ f(x)} = \lim_{d \to 0} \frac{cf(x+d) - cf(x)}{d}$$

$$c \lim_{d \to 0} \frac{f(x+d) - f(x)}{D}$$

$$= \mathbf{c} * \frac{d}{dx}\mathbf{f(x)}$$

$$\boxed{\frac{d}{dx}c\ f(x) = c\ \frac{d}{dx}f(x)}$$

25) (**Math** | **Calculus** | **Derivatives** | **Identities** | **Constant Rule**)

Proof of $\frac{d}{dx}c\ f(x) = c\ \frac{d}{dx}f(x)$ from the definition

We can use the definition of the derivative:

$$\frac{d}{dx}f(x) = \lim_{d \to 0} \frac{f(x+d)-f(x)}{d}$$

Therefore, $\frac{d}{dx}c\ f(x)$ can be written as such:

$$\frac{d}{dx}\mathbf{c\ f(x)} = \lim_{d \to 0} \frac{cf(x+d) - cf(x)}{d}$$

$$c \lim_{d \to 0} \frac{f(x+d) - f(x)}{d}$$

= **c** * $\frac{d}{dx}$**f(x)**

26)

Differentiation Identities

(**Math** | **Calculus** | **Derivatives** | **Identities**)

Definitions of the Derivative:

df / dx = lim (dx -> 0) (f(x+dx) - f(x)) / dx *(right sided)*
df / dx = lim (dx -> 0) (f(x) - f(x-dx)) / dx *(left sided)*
df / dx = lim (dx -> 0) (f(x+dx) - f(x-dx)) / (2dx) *(both sided)*

$\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$ **(Fundamental Theorem for Derivatives)**

$\frac{d}{dx}$c f(x) = c PROOF
$\frac{d}{dx}$f(x) *(c is a constant)*
[[

Proof: *Constant Rule*

$\frac{d}{dx}$c f(x) = c $\frac{d}{dx}$f(x)

(**Math** | **Calculus** | **Derivatives** | **Identities** | Constant Rule)

Proof of $\frac{d}{dx}$c f(x) = c $\frac{d}{dx}$f(x) from the definition

We can use the definition of the derivative:

$$\frac{d}{dx}f(x) = \lim_{d \to 0} \frac{f(x+d)-f(x)}{d}$$

Therefore, $\frac{d}{dx}$c f(x) can be written as such:

$$\frac{d}{dx}\mathbf{c\ f(x)} = \lim_{d \to 0} \frac{cf(x+d) - cf(x)}{d}$$

c lim f(x+d) - f(x)

$$\lim_{d \to 0} \frac{\quad}{d}$$

$$= c * \frac{d}{dx}f(x) \;]]$$

$\frac{d}{dx}(f(x) + g(x)) = \frac{d}{dx}f(x) + \frac{d}{dx}g(x)$

[[

Proof: *Sum Rule*

$$\frac{d}{dx}[f(x) + g(x)] = \frac{d}{dx}f(x) + \frac{d}{dx}g(x)$$

(**Math** | **Calculus** | **Derivatives** | **Identities** | Sum Rule)

Proof of $\frac{d}{dx}[f(x) + g(x)] = \frac{d}{dx}f(x) + \frac{d}{dx}g(x)$ from the definition

We can use the definition of the derivative:

$$\frac{d}{dx}f(x) = \lim_{d \to 0} \frac{f(x+d)-f(x)}{d}$$

Therefore, $\frac{d}{dx}[f(x) + g(x)]$ can be written as such:

$$\frac{d}{dx}[f(x) + g(x)] = \lim_{d \to 0} \frac{[f(x+d)+g(x+d)] - [f(x)+g(x)]}{d}$$

$$= \lim_{d \to 0} \left(\frac{[f(x+d)-f(x)]}{d} + \frac{[g(x+d)-g(x)]}{d} \right)$$

$$= \lim_{d \to 0} \frac{f(x+d)-f(x)}{d} + \lim_{d \to 0} \frac{g(x+d)-g(x)}{d}$$

]]

$\frac{d}{dx}f(g(x)) = \frac{d}{dg}f(g) * \frac{d}{dx}g(x)$ **(chain rule)**

[[

Chain Rule

(**Math** | **Calculus** | **Derivatives** | **Identities** | Chain Rule)

$\frac{d}{dx}$f(g(x)) = $\frac{d}{dg}$f(g) * $\frac{d}{dx}$g(x)

Proof of $\frac{d}{dx}$f(g(x)) = $\frac{d}{dg}$f(g) * $\frac{d}{dx}$g(x) from the definition

We can use the definition of the derivative:

$$\frac{d}{dx}f(x) = \lim_{d \to 0} \frac{f(x+d)-f(x)}{d}$$

Therefore, $\frac{d}{dx}$f(g(x)) can be written as such:

$\frac{d}{dx}$**f(g(x))** = df/dx = **(f(g(x+d) - f(g(x))/d**
df/dx * 1/(dg/dx) = [(f(g(x+d) - f(g(x))/d] * [d/(g(x+d) - g(x))]
= (f(g(x+d))-f(g(x)))/(g(x+d)-g(x)) = **df/dg**

df/dx = df/dg * dg/dx []

]]

$\frac{d}{dx}$f(x)g(x) = f'(x)g(x) + f(x)g '(x) **(product rule)**

$\frac{d}{dx}$f(x)/g(x) = (f '(x)g(x) - f(x)g '(x)) / g$^{\wedge 2}$(x) **(quotient rule)**

Partial Differentiation Identities

if f(x(r,s), y(r,s))

df / dr = df / dx * dx / DR + df / dy * dy / DR

df / ds = df / dx * dx / Ds + df / dy * dy / Ds

if f(x(r,s))

df / DR = df / dx * dx / DR

df / Ds = df / dx * dx / Ds

directional derivative

df(x,y) / d(Xi sub a) = f1(x,y) cos(a) + f2(x,y) sin(a)

(Xi sub a) = angle counterclockwise from pos. x axis.

27)

Integral Identities

(**Math** | **Calculus** | **Integrals** | Identities)

Formal Integral Definition:

$\int_a^b f(x)\,dx = \lim_{(d \to 0)} \sum_{(k=1..n)} f(X_{(k)})\,(x_{(k)} - x_{(k-1)})$ *when...*

$a = x_0 < x_1 < x_2 < \dots < x_n = b$

$d = \max(x_1-x_0, x_2-x_1, \dots, x_n - x_{(n-1)})$

$x_{(k-1)} <= X_{(k)} <= x_{(k)} \qquad k = 1, 2, \dots, n$

$\int_a^b F'(x)\,dx = F(b) - F(a)$ **(Fundamental Theorem for integrals of derivatives)**

$\int a\,f(x)\,dx = a\int f(x)\,dx$ *(if a is constant)*

$\int f(x) + g(x)\,dx = \int f(x)\,dx + \int g(x)\,dx$

$\int_a^b f(x)\,dx = \int f(x)\,dx \mid (a\ b)$

$\int_a^b f(x)\,dx + \int_b^c f(x)\,dx = \int_a^c f(x)\,dx$

$\int f(u)\,du/dx\,dx = \int f(u)\,du$ **(integration by substitution)**

28)

Series Convergence Tests

(**Math** | **Calculus** | **Series Expansions** | Convergence Tests)

Definition of Convergence and Divergence in Series

The n^{th} partial sum of the series $\sum_{n=1}^{\infty} a_n$ is given by $S_n = a_1 + a_2 + a_3 + \ldots + a_n$. If the sequence of these partial sums $\{S_n\}$ converges to L, then the sum of the series converges to L. If $\{S_n\}$ diverges, then the sum of the series diverges.

Operations on Convergent Series

If $\sum a_n = A$, and $\sum b_n = B$, then the following also converge as indicated:

$$\sum ca_n = cA$$
$$\sum (a_n + b_n) = A + B$$
$$\sum (a_n - b_n) = A - B$$

Alphabetical Listing of Convergence Tests

Absolute Convergence

If the series $\sum_{n=1}^{\infty} |a_n|$ converges, then the series $\sum_{n=1}^{\infty} a_n$ also converges.

Alternating Series Test

If for all n, a_n is positive, non-increasing (i.e. $0 < a_{n+1} <= a_n$), and approaching zero, then the alternating series

$\sum_{n=1}^{\infty} (-1)^n a_n$ and $\sum_{n=1}^{\infty} (-1)^{n-1} a_n$

both converge.

If the alternating series converges, then the remainder $R_N = S - S_N$ (where S is the exact sum of the infinite series and S_N is the sum of the first N terms of the series) is bounded by $|R_N| <= a_{N+1}$

Deleting the first N Terms

If N is a positive integer, then the series

$\sum_{n=1}^{\infty} a_n$ and $\sum_{n=N+1}^{\infty} a_n$

both converge or both diverge.

Direct Comparison Test

If $0 <= a_n <= b_n$ for all n greater than some positive integer N, then the following rules apply:

If $\sum_{n=1}^{\infty} b_n$ converges, then $\sum_{n=1}^{\infty} a_n$ converges.

If $\sum_{n=1}^{\infty} a_n$ diverges, then $\sum_{n=1}^{\infty} b_n$ diverges.

Geometric Series Convergence

The geometric series is given by

$$\sum_{n=0}^{\infty} a\,r^n = a + a\,r + a\,r^2 + a\,r^3 + \ldots$$

If $|r| < 1$ then the following geometric series converges to $a / (1 - r)$.

If $|r| >= 1$ then the above geometric series diverges.

Integral Test

If for all $n >= 1$, $f(n) = a_n$, and f is positive, continuous, and decreasing then

$\sum_{n=1}^{\infty} a_n$ and $\int_1^{\infty} a_n$

either both converge or both diverge.

If the above series converges, then the remainder $R_N = S - S_N$ (where S is the exact sum of the infinite series and S_N is the sum of the first N terms of the series) is bounded by $0< = R_N <= \int (N..\infty)\, f(x)\, dx$.

Limit Comparison Test

If $\lim (n\text{-->}\infty)\ (a_n / b_n) = L$,

where a_n, $b_n > 0$ and L is finite and positive,

then the series $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ either both converge or both diverge.

n^{th}-Term Test for Divergence

If the sequence $\{a_n\}$ does not converge to zero, then the series $\sum_{n=1}^{\infty} a_n$ diverges.

p-Series Convergence

The p-series is given by

$$\sum_{n=1}^{\infty} 1/n^p = 1/1^p + 1/2^p + 1/3^p + \ldots$$

where $p > 0$ by definition.

If $p > 1$, then the series converges.

If $0 < p <= 1$ then the series diverges.

Ratio Test

If for all n, $n \neq 0$, then the following rules apply:
Let $L = \lim (n \to \infty) | a_{n+1} / a_n |$.

If $L < 1$, then the series $\sum_{n=1}^{\infty} a_n$ converges.

If $L > 1$, then the series $\sum_{n=1}^{\infty} a_n$ diverges.
If $L = 1$, then the test in *inconclusive*.

Root Test

Let $L = \lim (n \to \infty) | a_n |^{1/n}$.

If $L < 1$, then the series $\sum_{n=1}^{\infty} a_n$ converges.

If $L > 1$, then the series $\sum_{n=1}^{\infty} a_n$ diverges.
If $L = 1$, then the test in *inconclusive*.

Taylor Series Convergence

If f has derivatives of all orders in an interval I centered at c, then the Taylor series converges as indicated:

$$\sum_{n=0}^{\infty} (1/n!)\, f^{(n)}(c)\, (x - c)^n = f(x)$$

if and only if $\lim (n \to \infty)$ RN = 0 for all x in I.
The remainder $R_N = S - S_N$ of the Taylor series (where S is the exact sum of the infinite series and S_N is the sum of the first N terms of the series) is equal to $(1/(n+1)!)\, f^{(n+1)}(z)\, (x - c)^{n+1}$, where z is some constant between x and c.

29)

Series Properties

(**Math** | **Calculus** | **Series Expansions** | Properties)

Semiformal Definition of a "Series":

A series $\sum_{n=a}^{b} a_n$ is the *indicated* sum of all values of a_n when n is set to each integer from a to b inclusive; namely, the indicated sum of the values $a_a + AA_{+1} + AA_{+2} + \ldots + a_{b-1} + a_b$.

Definition of the "Sum of the Series":

The "sum of the series" is the *actual result* when all the terms of the series are summed.

Note the difference: "1 + 2 + 3" is an example of a "series," but "6" is the actual "sum of the series."

Algebraic Definition:

$$\sum_{n=a}^{b} a_n = AA + AA_{+1} + AA_{+2} + \ldots + AB-1 + AB$$

Summation Arithmetic:

$$\sum_{n=a}^{b} c\, a_n = c \sum_{n=a}^{b} a_n$$ *(constant c)*

$$\sum_{n=a}^{b} a_n + \sum_{n=a}^{b} b_n = \sum_{n=a}^{b} a_n + b_n$$

$$\sum_{n=a}^{b} a_n - \sum_{n=a}^{b} b_n = \sum_{n=a}^{b} a_n - b_n$$

Summation Identities on the Bounds:

$$\sum_{n=a}^{b} a_n + \sum_{n=b+1}^{c} a_n = \sum_{n=a}^{c} a_n$$

$$\sum_{n=a}^{b} a_n = \sum_{n=a-c}^{b-c} a_{n+c}$$

$$\sum_{n=a}^{b} a_n = \sum_{n=a/c}^{b/c} a_{nc}$$

$$\sum_{n=a}^{b} a_n = \sum_{n=g(a)}^{g(b)} a_{g^{-1}(c)}$$

|
(similar relations exist for subtraction and division as generalized below for any operation g)
|

30)

Proof: *Sum Rule*

$$\frac{d}{dx}[f(x) + g(x)] = \frac{d}{dx}f(x) + \frac{d}{dx}g(x)$$

(**Math** | **Calculus** | **Derivatives** | **Identities** | Sum Rule)

Proof of $\frac{d}{dx}[f(x) + g(x)] = \frac{d}{dx}f(x) + \frac{d}{dx}g(x)$ from the definition

We can use the definition of the derivative:

$$\frac{d}{dx}f(x) = \lim_{d \to 0} \frac{f(x+d)-f(x)}{d}$$

Therefore, $\frac{d}{dx}[f(x) + g(x)]$ can be written as such:

$$\frac{d}{dx}[f(x) + g(x)] = \lim_{d \to 0} \frac{[f(x+d)+g(x+d)] - [f(x)+g(x)]}{d}$$

$$= \lim_{d \to 0} \left(\frac{[f(x+d)-f(x)]}{d} + \frac{[g(x+d)-g(x)]}{d} \right)$$

$$= \lim_{d \to 0} \frac{f(x+d)-f(x)}{d} + \lim_{d \to 0} \frac{g(x+d)-g(x)}{d}$$

31)

Table of Integrals

(**Math** | **Calculus** | **Integrals** | **Table Of**)

Power of x.

$\int x^n\, dx = x^{(n+1)} / (n+1) + C$ $(n \neq -1)$ Proof	$\int 1/x\, dx = \ln\lvert x\rvert + C$

Exponential / Logarithmic

$\int e^x\, dx = e^x + C$ Proof	$\int b^x\, dx = b^x / \ln(b) + C$ Proof, Tip!
$\int \ln(x)\, dx = x \ln(x) - x + C$ Proof	

Trigonometric

$\int \sin x\, dx = -\cos x + C$ Proof	$\int \csc x\, dx = - \ln\lvert \csc x + \cot x\rvert + C$ Proof


Young Scientist M.H.Rana, www.matherana.synthasite.com

$\int$ COs x *dx* = sin x + C Proof	$\int$ sec x *dx* = ln|sec x + tan x| + C Proof
$\int$ tan x *dx* = -ln|COs x| + C Proof	$\int$ cot x *dx* = ln|sin x| + C Proof

Trigonometric Result

$\int$ COs x *dx* = sin x + C Proof	$\int$ CSC x cot x *dx* = - CSC x + C Proof
$\int$ sin x *dx* = COs x + C Proof	$\int$ sec x tan x *dx* = sec x + C Proof
$\int \sec^2 x\, dx = \tan x + C$ Proof	$\int \csc^2 x\, dx = -\cot x + C$ Proof

Inverse Trigonometric

$\int \arcsin x\, dx = x \arcsin x + \sqrt{(1-x^2)} + C$
$\int \text{arccsc}\, x\, dx = x \arccos x - \sqrt{(1-x^2)} + C$
$\int \arctan x\, dx = x \arctan x - (1/2) \ln(1+x^2) + C$

Inverse Trigonometric Result

$\int \frac{dx}{\sqrt{(1 - x^2)}} = \arcsin x + C$	**Useful Identities** arccos x = π/2 - arcsin x (-1 <= x <= 1) arccsc x = π/2 - arcsec x (|x| >= 1) arccot x = π/2 - arctan x (for all x)
$\int \frac{dx}{x\sqrt{(x^2 - 1)}} = \text{arcsec}\|x\| + C$	
$\int$ *dx* = arctan x + C	

$1 + x^2$

Hyperbolic

$\int \sinh x \, dx = \cosh x + C$ Proof	$\int \operatorname{csch} x \, dx = \ln \lvert\tanh(x/2)\rvert + C$ Proof
$\int \cosh x \, dx = \sinh x + C$ Proof	$\int \operatorname{sech} x \, dx = \arctan (\sinh x) + C$
$\int \tanh x \, dx = \ln (\cosh x) + C$ Proof	$\int \coth x \, dx = \ln \lvert\sinh x\rvert + C$ Proof

Click on Proof for a proof/discussion of a theorem.

To solve a more complicated integral, see The Integrator at http://integrals.wolfram.com/

32)

Chain Rule

(**Math** | **Calculus** | **Derivatives** | **Identities** | Chain Rule)

$$\frac{d}{dx}f(g(x)) = \frac{d}{dg}f(g) * \frac{d}{dx}g(x)$$

Proof of $\frac{d}{dx}f(g(x)) = \frac{d}{dg}f(g) * \frac{d}{dx}g(x)$ from the definition

We can use the definition of the derivative:

$$\frac{d}{dx}f(x) = \lim_{d \to 0} \frac{f(x+d)-f(x)}{d}$$

Therefore, $\frac{d}{dx}f(g(x))$ can be written as such:

$\frac{d}{dx}$**f(g(x))** = df/dx = **(f(g(x+d) - f(g(x))/d**
df/dx * 1/(dg/dx) = [(f(g(x+d) - f(g(x))/d] * [d/(g(x+d) - g(x))]
= (f(g(x+d))-f(g(x)))/(g(x+d)-g(x)) = **df/dg**

df/dx = df/dg * dg/dx

33)

Derivatives: Min, Max, Critical Points...

(**Math** | **Calculus** | **Derivatives** | **Extrema / Concavity / Other**)

Asymptotes

Definition of a horizontal asymptote: The line $y = y_0$ is a "horizontal asymptote" of f(x) if and only if f(x) approaches y_0 as x approaches + or - ∞.

Definition of a vertical asymptote: The line $x = x_0$ is a "vertical asymptote" of f(x) if and only if f(x) approaches + or - ∞ as x approaches x_0 from the left or from the right.

Definition of a slant asymptote: the line y = ax + b is a "slant asymptote" of f(x) if and only if $\lim_{(x \to +/-\infty)} f(x) = ax + b$.

Concavity

Definition of a concave up curve: f(x) is "concave up" at x_0 if and only if f '(x) is increasing at x_0

Definition of a concave down curve: f(x) is "concave down" at x_0 if and only if f '(x) is decreasing at x_0

The second derivative test: If f ''(x) exists at x_0 and is positive, then f ''(x) is concave up at x_0. If f ''(x_0) exists and is negative, then f(x) is concave down at x_0. If f ''(x) does not exist or is zero, then the test fails.

Critical Points

Definition of a critical point: a critical point on f(x) occurs at x_0 if and only if either f '(x_0) is zero or the derivative doesn't exist.

Extrema (Maxima and Minima)

Local (Relative) Extrema

Definition of a local maxima: A function f(x) has a local maximum at x_0 if and only if there exists some interval I containing x_0 such that f(x_0) >= f(x) for all x in I.

Definition of a local minima: A function f(x) has a local minimum at x_0 if and only if there exists some interval I containing x_0 such that f(x_0) <= f(x) for all x in I.

Occurrence of local extrema: All local extrema occur at critical points, but not all critical points occur at local extrema.

The first derivative test for local extrema: If f(x) is increasing (f '(x) > 0) for all x in some interval (a, x_0] and f(x) is decreasing (f '(x) < 0) for all x in some interval [x_0, b), then f(x) has a local maximum at x_0. If f(x) is decreasing (f '(x)

< 0) for all x in some interval (a, x_0] and f(x) is increasing (f '(x) > 0) for all x in some interval [x_0, b), then f(x) has a local minimum at x_0.

The second derivative test for local extrema: If f '(x_0) = 0 and f ''(x_0) > 0, then f(x) has a local minimum at x_0. If f '(x_0) = 0 and f ''(x_0) < 0, then f(x) has a local maximum at x_0.

Absolute Extrema

Definition of absolute maxima: y_0 is the "absolute maximum" of f(x) on I if and only if y_0 >= f(x) for all x on I.

Definition of absolute minima: y_0 is the "absolute minimum" of f(x) on I if and only if y_0 <= f(x) for all x on I.

The extreme value theorem: If f(x) is continuous in a closed interval I, then f(x) has at least one absolute maximum and one absolute minimum in I.

Occurrence of absolute maxima: If f(x) is continuous in a closed interval I, then the absolute maximum of f(x) in I is the maximum value of f(x) on all local maxima and endpoints on I.

Occurrence of absolute minima: If f(x) is continuous in a closed interval I, then the absolute minimum of f(x) in I is the minimum value of f(x) on all local minima and endpoints on I.

Alternate method of finding extrema: If f(x) is continuous in a closed interval I, then the absolute extrema of f(x) in I occur at the critical points and/or at the endpoints of I.
(This is a less specific form of the above.)

Increasing/Decreasing Functions

Definition of an increasing function: A function f(x) is "increasing" at a point x_0 if and only if there exists some interval I containing x_0 such that f(x_0) > f(x) for all x in I to the left of x_0 and f(x_0) < f(x) for all x in I to the right of x_0.

Definition of a decreasing function: A function f(x) is "decreasing" at a point x_0 if and only if there exists some interval I containing x_0 such that f(x_0) < f(x) for all x in I to the left of x_0 and f(x_0) > f(x) for all x in I to the right of x_0.

The first derivative test: If f '(x_0) exists and is positive, then f '(x) is increasing at x_0. If f '(x) exists and is negative, then f(x) is decreasing at x_0. If f '(x_0) does not exist or is zero, then the test tells fails.

Inflection Points

Definition of an inflection point: An inflection point occurs on f(x) at x_0 if and only if f(x) has a tangent line at x_0 and there exists and interval I containing x_0 such that f(x) is concave up on one side of x_0 and concave down on the other side.

34)

Series Expansions

(**Math** | **Calculus** | Series Expansions)

Series (Summation) Expansions

Basic Properties
Convergence Tests

***Function-->Summation* and *Summation-->Function* Conversions**

Select function **f(x)** to expand into a summation $f(x) = \sum ?$	Select term a_n in summation to simplify: $\sum a_n = ?$
Exponential / Logarithm Functions f(x) = e; e^{-1}; e^{x} f(x) = ln(x) **Root Functions** f(x) = $\sqrt{x}$; $1/\sqrt{x}$	**Geometric Series** $a_n = r^{n}$ **Power Series** a_n = n; n^{2}; n^{3}; ... a_n = 1/n; $1/n^{2}$; $1/n^{3}$; $1/n^{4}$; $1/n^{5}$; $1/n^{6}$; $1/n^{7}$; $1/n^{8}$; $1/n^{9}$; $1/n^{10}$; $1/n^{p}$

35)

Special Functions

(Math | Calculus | Integrals | Special Functions)

Some of these functions I have seen defined under both intervals (0 to x) and (x to inf). In that case, both ***variant*** definitions are listed.

gamma = Euler's γ constant = 0.5772156649...

$\Gamma(x)$ = Gamma(x) = $\int_0^{\infty} t^{(x-1)} e^{(-t)} dt$ (Gamma function)

B(x,y) = $\int_0^{1} t^{(x-1)} (1-t)^{(y-1)} DT$ (Beta function)

Ei(x) = $\int_x^{\infty} e^{(-t)}/t\ DT$ (exponential integral) or it's variant, *NONEQUIVALENT* form:

Ei(x) = γ + ln(x) + $\int_0^{x} (e^{t} - 1)/t\ DT$ = gamma + ln(x) + $\sum$(n=1..inf)x^{n}/(n*n!)

li(x) = $\int_2^{x} 1/\ln(t)\ DT$ (logarithmic integral)

Si(x) = $\int_x^{\infty} \sin(t)/t\ DT$ (sine integral) or it's variant, *NONEQUIVALENT* form:

Si(x) = $\int_0^{x} \sin(t)/t\ DT$ = PI/2 - $\int_x^{\infty} \sin(t)/t\ DT$

Ci(x) = $\int_x^{\infty} \cos(t)/t\ DT$ (cosine integral) or it's variant, *NONEQUIVALENT* form:

CI(x) = - $\int_x^{\infty} \cos(t)/t\ DT$ = gamma + ln(x) + $\int_0^{x} (\cos(t) - 1)/t\ DT$ (cosine integral)

Chi(x) = gamma + ln(x) + $\int_0^x$ **(cosh(t)-1) / t DT (hyperbolic cosine integral)**

Shi(x) = $\int_0^x$ **sinh(t) / t DT (hyperbolic sine integral)**

Erf(x) = 2 / PI^(1/2) $\int_0^x e^{\wedge(-t\wedge 2)}$ **DT = 2 /** $\sqrt{}$**PI** $\sum$**(n=0..inf) (-1)^n x^(2n+1) / (n! (2n+1)) (error function)**

FresnelC(x) = $\int_0^x$ **COs(PI / 2 t^2) DT**

FresnelS(x) = $\int_0^x$ **sin(PI / 2 t^2) DT**

dilog(x) = $\int_1^x$ **ln(t) / (1-t) DT**

Psi(x) = $\frac{d}{dx}$**ln(Gamma(x))**
Psi(n,x) = n^{th} **derivative of Psi(x)**
W(x) = inverse of x*e^x
L $_{\text{sub } n}$ **(x) = (e^x / n!)(x^n e^(-x))** $^{(n)}$ **(laguerre polynomial degree n. (n) meaning** n^{th} **derivative)**

Zeta(s) = $\sum$**(n=1..inf) 1 / n^s**

Dirichlet's beta function B(x) = $\sum$**(n=0..inf) (-1)^n / (2n+1)^x**

Theorems with hyperlinks have proofs, related theorems, discussions, and/or other info.

36)

Table of Derivatives

(**Math** | **Calculus** | **Derivatives** | **Table Of**)

Power of x.

$\frac{d}{dx}c = 0$	$\frac{d}{dx}x = 1$	$\frac{d}{dx}x^n = n\, x^{(n-1)}$ Proof

Exponential / Logarithmic

$\frac{d}{dx}e^x = e^x$ Proof	$\frac{d}{dx}b^x = b^x \ln(b)$ Proof	$\frac{d}{dx}\ln(x) = 1/x$ Proof

Trigonometric

$\frac{d}{dx}\sin x = \cos x$ Proof	$\frac{d}{dx}\csc x = -\csc x \cot x$ Proof
$\frac{d}{dx}\cos x = -\sin x$ Proof	$\frac{d}{dx}\sec x = \sec x \tan x$ Proof
$\frac{d}{dx}\tan x = \sec^2 x$ Proof	$\frac{d}{dx}\cot x = -\csc^2 x$ Proof

Inverse Trigonometric

$\frac{d}{dx}\arcsin x = \frac{1}{\sqrt{(1 - x^2)}}$	$\frac{d}{dx}\operatorname{arccsc} x = \frac{-1}{\lvert x\rvert \sqrt{(x^2 - 1)}}$
$\frac{d}{dx}\arccos x = \frac{-1}{\sqrt{(1 - x^2)}}$	$\frac{d}{dx}\operatorname{arcsec} x = \frac{1}{\lvert x\rvert \sqrt{(x^2 - 1)}}$
$\frac{d}{dx}\arctan x = \frac{1}{1 + x^2}$	$\frac{d}{dx}\operatorname{arccot} x = \frac{-1}{1 + x^2}$

Hyperbolic

$\frac{d}{dx}\sinh x = \cosh x$ Proof	$\frac{d}{dx}\operatorname{csch} x = -\coth x \operatorname{csch} x$ Proof
$\frac{d}{dx}\cosh x = \sinh x$ Proof	$\frac{d}{dx}\operatorname{sech} x = -\tanh x \operatorname{sech} x$ Proof
$\frac{d}{dx}\tanh x = 1 - \tanh^2 x$ Proof	$\frac{d}{dx}\coth x = 1 - \coth^2 x$ Proof

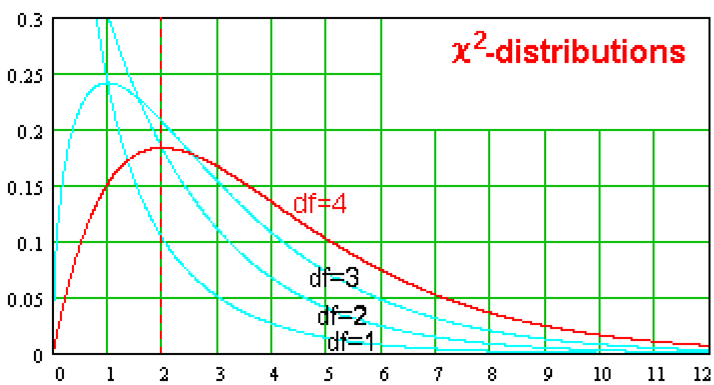
Those with hyperlinks have proofs.

Statistic

37)

chi^2-distribution / b >

(Math | Distributions | chi^2-Distributions)



The χ^2-distribution, with n degrees of freedom, is given by the equation:

$f(\chi^2) = (\chi^2)^{(n/2 - 1)} e^{(-\chi^2 / 2)} 2^{(-n/2)} / \Gamma(n/2)$

The area within an interval (a, ∞) = $\int_a^\infty f(\chi^2)\, d\chi^2 = \Gamma(n/2, a/2) / \Gamma(n/2)$ *(See also Gamma function)*

38)

Normal Probability Calculator

(Math | Statistics | z-Distribution | Probability Calculator)

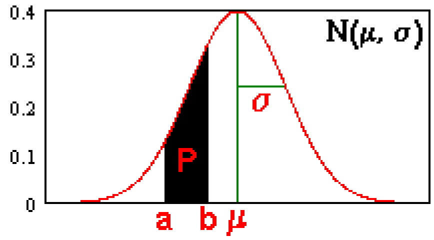
Java Normal Probability Calculator

(for Microsoft 2.0+/Netscape 2.0+/Java Script browsers only)
To find the area **P** under the normal probability curve N(mean, standard_deviation) within the interval (left, right), type in the 4 parameters and press "Calculate". *The standard normal curve*

Young Scientist M.H.Rana, www.matherana.synthasite.com

N(0,1) has a mean=0 and s.d.=1. Use -inf and +inf for infinite limits.

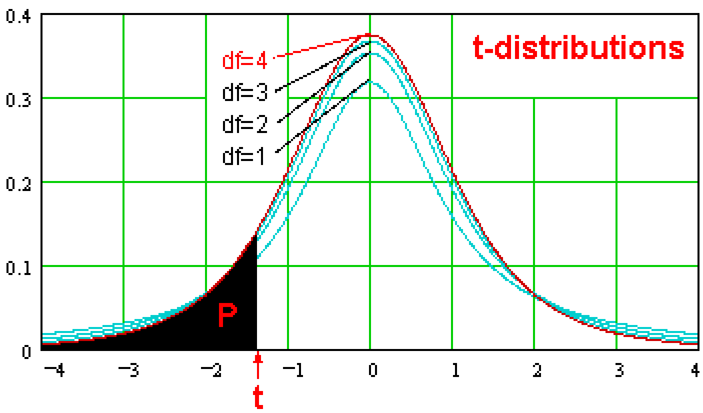
left bound	right bound	mean	standard deviation
-inf	0	0	1



39)

t-distributions / b>

(**Math** | **Distributions** | t-Distributions)



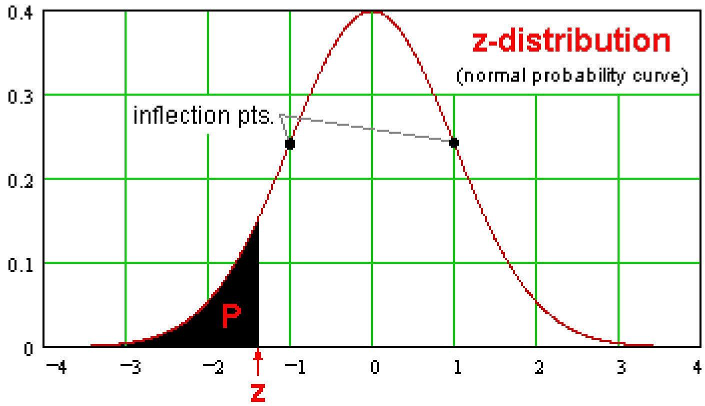
The t-distribution, with n degrees of freedom, is given by the equation:

f(t) = [Γ((n + 1)/2) (1 + $x^{\wedge 2}$ / n)$^{\wedge(-n/2 - 1/2)}$] / [Γ(n/2) √(PI n)] *(See also Gamma Function.)*

40)

z-distribution / b>

(**Math** | **Distributions** | z-Distribution)



The z- is a N(0, 1) distribution, given by the equation:

$f(z) = 1/\sqrt{(2PI)}\ e^{(-z^2/2)}$

The area within an interval (a,b) = normalcdf(a,b) = $\int_a^b e^{-z^2/2}\,dz$ *(It is not integrable algebraically.)*

The Taylor expansion of the above assists in speeding up the calculation:

$$\text{normalcdf}(-\infty, z) = 1/2 + 1/\sqrt{(2PI)} \sum_{(k=0..\infty)} \left[\left((-1)^{k}\, x^{(2k+1)} \right) / \left((2k+1)\, 2^{k}\, k! \right) \right]$$

Standard Normal Probabilities:

(The table is based on the area **P** under the standard normal probability curve, below the respective **z**-statistic.)

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-4.0	0.00003	0.00003	0.00003	0.00003	0.00003	0.00003	0.00002	0.00002	0.00002	0.00002
-3.9	0.00005	0.00005	0.00004	0.00004	0.00004	0.00004	0.00004	0.00004	0.00003	0.00003
-3.8	0.00007	0.00007	0.00007	0.00006	0.00006	0.00006	0.00006	0.00005	0.00005	0.00005
-3.7	0.00011	0.00010	0.00010	0.00010	0.00009	0.00009	0.00008	0.00008	0.00008	0.00008
-3.6	0.00016	0.00015	0.00015	0.00014	0.00014	0.00013	0.00013	0.00012	0.00012	0.00011
-3.5	0.00023	0.00022	0.00022	0.00021	0.00020	0.00019	0.00019	0.00018	0.00017	0.00017
-3.4	0.00034	0.00032	0.00031	0.00030	0.00029	0.00028	0.00027	0.00026	0.00025	0.00024
-3.3	0.00048	0.00047	0.00045	0.00043	0.00042	0.00040	0.00039	0.00038	0.00036	0.00035
-3.2	0.00069	0.00066	0.00064	0.00062	0.00060	0.00058	0.00056	0.00054	0.00052	0.00050
-3.1	0.00097	0.00094	0.00090	0.00087	0.00084	0.00082	0.00079	0.00076	0.00074	0.00071
-3.0	0.00135	0.00131	0.00126	0.00122	0.00118	0.00114	0.00111	0.00107	0.00103	0.00100
-2.9	0.00187	0.00181	0.00175	0.00169	0.00164	0.00159	0.00154	0.00149	0.00144	0.00139
-2.8	0.00256	0.00248	0.00240	0.00233	0.00226	0.00219	0.00212	0.00205	0.00199	0.00193
-2.7	0.00347	0.00336	0.00326	0.00317	0.00307	0.00298	0.00289	0.00280	0.00272	0.00264
-2.6	0.00466	0.00453	0.00440	0.00427	0.00415	0.00402	0.00391	0.00379	0.00368	0.00357
-2.5	0.00621	0.00604	0.00587	0.00570	0.00554	0.00539	0.00523	0.00508	0.00494	0.00480
-2.4	0.00820	0.00798	0.00776	0.00755	0.00734	0.00714	0.00695	0.00676	0.00657	0.00639

-2.3	0.01072	0.01044	0.01017	0.00990	0.00964	0.00939	0.00914	0.00889	0.00866	0.00842
-2.2	0.01390	0.01355	0.01321	0.01287	0.01255	0.01222	0.01191	0.01160	0.01130	0.01101
-2.1	0.01786	0.01743	0.01700	0.01659	0.01618	0.01578	0.01539	0.01500	0.01463	0.01426
-2.0	0.02275	0.02222	0.02169	0.02118	0.02067	0.02018	0.01970	0.01923	0.01876	0.01831
-1.9	0.02872	0.02807	0.02743	0.02680	0.02619	0.02559	0.02500	0.02442	0.02385	0.02330
-1.8	0.03593	0.03515	0.03438	0.03362	0.03288	0.03216	0.03144	0.03074	0.03005	0.02938
-1.7	0.04456	0.04363	0.04272	0.04181	0.04093	0.04006	0.03920	0.03836	0.03754	0.03673
-1.6	0.05480	0.05370	0.05262	0.05155	0.05050	0.04947	0.04846	0.04746	0.04648	0.04551
-1.5	0.06681	0.06552	0.06425	0.06301	0.06178	0.06057	0.05938	0.05821	0.05705	0.05592
-1.4	0.08076	0.07927	0.07780	0.07636	0.07493	0.07353	0.07214	0.07078	0.06944	0.06811
-1.3	0.09680	0.09510	0.09342	0.09176	0.09012	0.08851	0.08691	0.08534	0.08379	0.08226
-1.2	0.11507	0.11314	0.11123	0.10935	0.10749	0.10565	0.10383	0.10204	0.10027	0.09852
-1.1	0.13566	0.13350	0.13136	0.12924	0.12714	0.12507	0.12302	0.12100	0.11900	0.11702
-1.0	0.15865	0.15625	0.15386	0.15150	0.14917	0.14686	0.14457	0.14231	0.14007	0.13786
-0.9	0.18406	0.18141	0.17878	0.17618	0.17361	0.17105	0.16853	0.16602	0.16354	0.16109
-0.8	0.21185	0.20897	0.20611	0.20327	0.20045	0.19766	0.19489	0.19215	0.18943	0.18673
-0.7	0.24196	0.23885	0.23576	0.23269	0.22965	0.22663	0.22363	0.22065	0.21769	0.21476
-0.6	0.27425	0.27093	0.26763	0.26434	0.26108	0.25784	0.25462	0.25143	0.24825	0.24509
-0.5	0.30853	0.30502	0.30153	0.29805	0.29460	0.29116	0.28774	0.28434	0.28095	0.27759
-0.4	0.34457	0.34090	0.33724	0.33359	0.32997	0.32635	0.32276	0.31917	0.31561	0.31206
-0.3	0.38209	0.37828	0.37448	0.37070	0.36692	0.36317	0.35942	0.35569	0.35197	0.34826
-0.2	0.42074	0.41683	0.41293	0.40904	0.40516	0.40129	0.39743	0.39358	0.38974	0.38590
-0.1	0.46017	0.45620	0.45224	0.44828	0.44433	0.44038	0.43644	0.43250	0.42857	0.42465
-0.0	0.50000	0.49601	0.49202	0.48803	0.48404	0.48006	0.47607	0.47209	0.46811	0.46414

Java Normal Probability Calculator

41)

Fourier Series

(**Math** | **Advanced** | Fourier Series)

- **The fourier series of the function f(x)**

a(0) / 2 + $\sum$**(k=1..∞) (a(k) cos kx + b(k) sin kx)**

a(k) = 1/PI $\int_{-\pi}^{\pi}$ **f(x) cos kx dx**

b(k) = 1/PI $\int_{-\pi}^{\pi}$ **f(x) sin kx dx**

- **Remainder of fourier series**. Sn(x) = sum of first n+1 terms at x.

remainder(n) = f(x) - Sn(x) = $1/PI \int_{-\pi}^{\pi} f(x+t)\, Dn(t)\, dt$

Sn(x) = $1/PI \int_{-\pi}^{\pi} f(x+t)\, Dn(t)\, dt$

Dn(x) = Dirichlet kernel = 1/2 + cos x + cos 2x + .. + cos nx = [sin(n + 1/2)x] / [2sin(x/2)]

Riemann's Theorem. If f(x) is continuous except for a finite # of finite jumps in every finite interval then:

$$\lim_{(k\to\infty)} \int_{a}^{b} f(t) \cos kt\, dt = \lim_{(k\to\infty)} \int_{a}^{b} f(t) \sin kt\, dt = 0$$

The fourier series of the function f(x) in an arbitrary interval.

$$A(0) / 2 + \sum_{(k=1..\infty)} [A(k) \cos (k(PI)x / m) + B(k) (\sin k(PI)x / m)]$$

$$a(k) = 1/m \int_{-m}^{m} f(x) \cos (k(PI)x / m)\, dx$$

$$b(k) = 1/m \int_{-m}^{m} f(x) \sin (k(PI)x / m)\, dx$$

Parseval's Theorem. If f(x) is continuous; f(-PI) = f(PI) then

$$1/PI \int_{-\pi}^{\pi} f^{2}(x)\, dx = a(0)^{2} / 2 + \sum_{(k=1..\infty)} (a(k)^{2} + b(k)^{2})$$

Fourier Integral of the function f(x)

$$f(x) = \int_{0}^{\infty} (a(y) \cos yx + b(y) \sin yx)\, dy$$

$$a(y) = 1/PI \int_{-\infty}^{\infty} f(t) \cos ty\, dt$$

$$b(y) = 1/PI \int_{-\infty}^{\infty} f(t) \sin ty\, dt$$

$$f(x) = 1/PI \int_{0}^{\infty} dy \int_{-\infty}^{\infty} f(t) \cos (y(x-t))\, dt$$

Special Cases of Fourier Integral

if f(x) = f(-x) then

$$f(x) = 2/PI \int_0^{\infty} \cos xy \, dy \int_0^{\infty} f(t) \cos yt \, dt$$

if f(-x) = -f(x) then

$$f(x) = 2/PI \int_0^{\infty} \sin xy \, dy \int_0^{\infty} \sin yt \, dt$$

Fourier Transforms

Fourier Cosine Transform

$$g(x) = \sqrt{(2/PI)} \int_0^{\infty} f(t) \cos xt \, dt$$

Fourier Sine Transform

$$g(x) = \sqrt{(2/PI)} \int_0^{\infty} f(t) \sin xt \, dt$$

Identities of the Transforms

If f(-x) = f(x) then

Fourier Cosine Transform (Fourier Cosine Transform (f(x))) = f(x)

If f(-x) = -f(x) then
Fourier Sine Transform (Fourier Sine Transform (f(x))) = f(x)

42)

Recursive Formulas

(**Math** | **Advanced Topics** | **Recursive Formulas)**

Recursive Formulas

Recursive expansions are given for the following functions.

- $y^{1/n}$ open
- B / A open
- $\sqrt{(x)}$ open

43)

Recursive Formulas

(**Math** | **Advanced Topics** | **Recursive Formulas** | **B / A)**

Recursive Formulas for B / A

Explicit form:

Find B/A where B and A are real numbers and B > 0

Recursive form:
Convert B and A to scientific notation base 2 (C++ has function "frexp" for this). Note: mantissa is ^{3} .5 and < 1.

Let

a = mantissa of A
b = mantissa of B
exp = exponent of b - exponent of a
$x_0 = 1$
$x_{n+1} = x_n(2 - a\ x_n)$

Reiterate x until desired precision reached. Result only has to be close, not perfect. I suggest about 5 times.
$y_0 = b\ x_n$
$y_{i+1} = y_i + x_n(b - a\ y_i)$
Reiterate y until desired precision reached.

$B / A = y_i * 2^{exp}$

Example: 314.51 / 5.6789
Written in base-2 scientific notation:
$B = 0.61357421875 * 2^9$
$A = 0.7098625 * 2^3$
exp = 9 - 3 = 6

iteration	**value**
x_0	1
x_1	1.2901375
x_2	1.398740976607287
x_3	1.408652781763906
x_4	1.408723516847958 (will use this value for x)
iteration	**value**
y_0	0.864356431284738
y_1	0.864356433463227
y_2	0.864356433463227
y_3	0.864356433463227
y_4	0.864356433463227
y_5	0.864356433463227

$B / A = 0.864356433463227 * 2^6 =$
55.318811741710538

55.318811741710538 (true value)

Source: Jeff Yates, et al.

44)

Recursive Formulas

(**Math** | **Advanced** | **Recursive Formulas** | √(A))

Recursive Formulas for √(A)

Explicit form:

Find √(A), where A is a real number > 0.

Recursive form:
Convert A to scientific notation base 2 (C++ has function "frexp" for this). Note: mantissa is ³ .5 and < 1.

Let

a = mantissa of A
exp = exponent of a
$a = a * 2^{(exp \bmod 2)}$
exp = exp \ 2 (Note: integer divide)
$x_{n+1} = (x_n/2)(3 - ax_n^2)$

Reiterate about 5 or 6 times then do y with that result.
$y_{i+1} = y_i + (x_n/2)(a - y_i^2)$
Reiterate until required precision attained.

$\sqrt{A} = y_i * 2^{exp}$

Source: Jeff Yates.

45)

Recursive Formulas

(**Math** | **Advanced** | **Recursive Formulas** | $y^{1/n}$)

Recursive Formulas for $y^{1/n}$

Explicit form:

$y^{1/n} = x$

Recursive form:
$x_{k+1} = (x_k + y / (x_k)^{n-1}) / 2$

where y ³ 0 and n > 0
or y Î Â and n is odd, positive, and integer.
(for negative n, evaluate the above formula with n positive, then invert your answer).

Example: $2^{1/3} = 1.259921049894...$

iteration	value
x_0	1.0000000000
x_1	1.5000000000
x_2	1.1944444444
x_3	1.2981416381
x_4	1.2424821566
x_5	1.2690093603
x_6	1.2554742937
x_7	1.2621680807
x_8	1.2588035314
x_9	1.2604812976
x_{10}	1.2596412994
x_{20}	1.2599207765
x_{50}	1.259921049894
x_∞	**1.259921049894...**

Source: Jeff Yates, et al.

46)

Transforms

(**Math** | **Advanced** | **Transforms**)

Laplace Transforms

$f(x) = \int_0^\infty e^{(-xt)}\, g(t)\, dt$ **(Laplace Transform)**

$f(x) = \int_0^\infty e^{(-xt)}\, g(t)\, d\alpha(t)$ **(Laplace-Stieltjes Transform)**

$f2(x) = L\{L\{g(t)\}\} = \int_0^\infty g(t)/(x+t)\, dt$ **(Stieltjes Transform)**

Fourier Transforms

$f(x) = 1/\sqrt{(2\pi)} \int_{-\infty}^{\infty} g(t)\, e^{(i\,tx)}\, dt$ **(Fourier Transform)**

$f(x) = \sqrt{(2/\pi)} \int_0^\infty g(x) \cos(xt)\, dt$ **(Cosine Transform)**

$f(x) = \sqrt{(2/\pi)} \int_0^\infty g(x) \sin(xt)\, dt$ **(Sine Transform)**

Power Series Transform

$$f(x) = \sum_{(k=0..\infty)} g(k)\, x^{\wedge k}$$

47)

Addition Table / b>

(**Math** | **General** | AdditionTable)

Addition Table

+	0	1	2	3	4	5	6	7	8	9	10
0	0	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10	11
2	2	3	4	5	6	7	8	9	10	11	12
3	3	4	5	6	7	8	9	10	11	12	13
4	4	5	6	7	8	9	10	11	12	13	14
5	5	6	7	8	9	10	11	12	13	14	15
6	6	7	8	9	10	11	12	13	14	15	16
7	7	8	9	10	11	12	13	14	15	16	17
8	8	9	10	11	12	13	14	15	16	17	18
9	9	10	11	12	13	14	15	16	17	18	19
10	10	11	12	13	14	15	16	17	18	19	20

48)

E

(**Math** | **Miscellaneous** | **Constants** | e)

e = 2.7182818284 5904523536 0287471352 6624977572 4709369995 9574966967 6277240766 3035354759 4571382178 5251664274 ...

$e = \lim_{(n \to 0)} (1 + n)^{\wedge(1/n)}$ **or** $e = \lim_{(n \to \infty)} (1 + 1/n)^{\wedge n}$

$e = \sum_{n=0}^{\infty} 1 / k!$

see also Exponential Function Expansions.

49)

Fraction to Decimal Conversion

(**Math** | **General** | **Fraction to Decimal Conversion**)

Fraction to Decimal Conversion Tables

Important Note: any span of numbers that is underlined signifies that those numbers are repeated. For example, $0.\underline{09}$ signifies 0.090909....

Only fractions in lowest terms are listed. For instance, to find 2/8, first simplify it to 1/4 then search for it in the table below.

fraction = decimal			
1 / 1 = 1			
1 / 2 = 0.5			
1 / 3 = $0.\underline{3}$	2/3 = $0.\underline{6}$		
1 / 4 = 0.25	3/4 = 0.75		
1 / 5 = 0.2	2/5 = 0.4	3/5 = 0.6	4/5 = 0.8
1 / 6 = $0.1\underline{6}$	5/6 = $0.8\underline{3}$		
1 / 7 = $0.\underline{142857}$	2/7 = $0.\underline{285714}$	3/7 = $0.\underline{428571}$	4/7 = $0.\underline{571428}$
	5/7 = $0.\underline{714285}$	6/7 = $0.\underline{857142}$	
1 / 8 = 0.125	3/8 = 0.375	5/8 = 0.625	7/8 = 0.875
1 / 9 = $0.\underline{1}$	2/9 = $0.\underline{2}$	4/9 = $0.\underline{4}$	5/9 = $0.\underline{5}$
	7/9 = $0.\underline{7}$	8/9 = $0.\underline{8}$	
1 / 10 = 0.1	3/10 = 0.3	7/10 = 0.7	9/10 = 0.9
1 / 11 = $0.\underline{09}$	2/11 = $0.\underline{18}$	3/11 = $0.\underline{27}$	4/11 = $0.\underline{36}$
	5/11 = $0.\underline{45}$	6/11 = $0.\underline{54}$	7/11 = $0.\underline{63}$
	8/11 = $0.\underline{72}$	9/11 = $0.\underline{81}$	10/11 = $0.\underline{90}$

1 / 12 = 0.083	5/12 = 0.416	7/12 = 0.583	11/12 = 0.916
1 / 16 = 0.0625	3/16 = 0.1875	5/16 = 0.3125	7/16 = 0.4375
	11/16 = 0.6875	13/16 = 0.8125	15/16 = 0.9375
1 / 32 = 0.03125	3/32 = 0.09375	5/32 = 0.15625	7/32 = 0.21875
	9/32 = 0.28125	11/32 = 0.34375	13/32 = 0.40625
	15/32 = 0.46875	17/32 = 0.53125	19/32 = 0.59375
	21/32 = 0.65625	23/32 = 0.71875	25/32 = 0.78125
	27/32 = 0.84375	29/32 = 0.90625	31/32 = 0.96875

Need to convert a repeating decimal to a fraction? Follow these examples:
Note the following pattern for repeating decimals:
0.22222222... = 2/9
0.54545454... = 54/99
0.298298298... = 298/999
Division by 9's causes the repeating pattern.

Note the pattern if zeros precede the repeating decimal:
0.022222222... = 2/90
0.00054545454... = 54/99000
0.00298298298... = 298/99900
Adding zero's to the denominator adds zero's before the repeating decimal.

To convert a decimal that begins with a non-repeating part, such as 0.21456456456456456..., to a fraction, write it as the sum of the non-repeating part and the repeating part.
0.21 + 0.00456456456456456...
Next, convert each of these decimals to fractions. The first decimal has a divisor of power ten. The second decimal (which repeats) is converted according to the pattern given above.
21/100 + 456/99900
Now add these fraction by expressing both with a common divisor
20979/99900 + 456/99900
and add.
21435/99900
Finally simplify it to lowest terms
1429/6660
and check on your calculator or with long division.
= 0.2145645645...

50)

Gamma Constant

(**Math** | **Miscellaneous** | **Constants** | Gamma)

gamma = γ = 0.5772156649 0153286061 ...

$\gamma = \lim_{(n \to \infty)} \left(1 + 1/2 + 1/3 + 1/4 + \dots + 1/n - \ln(n) \right) = 0.5772156649\dots$

$\gamma = -\int_0^\infty e^{-x} \ln x \, dx$

$\Gamma'(1) = -\gamma$ *(see Gamma Function)*

51)

Interest and Exponential Growth

(Math | General | Interest and Exponential Growth)

The Compound Interest Equation

$P = C (1 + r/n)^{nt}$

where
- P = future value
- C = initial deposit
- r = interest rate (expressed as a fraction: eg. 0.06)
- n = # of times per year interest is compounded
- t = number of years invested

Simplified Compound Interest Equation

When interest is only compounded once per year (n=1), the equation simplifies to:
$P = C (1 + r)^t$

Continuous Compound Interest

When interest is compounded continually (i.e. n --> ∞), the compound interest equation takes the form:
$P = C e^{rt}$

Demonstration of Various Compounding

The following table shows the final principal (P), after t = 1 year, of an account initially with C = $10000, at 6% interest rate, with the given compounding (n). As is shown, the method of compounding has little effect.

N	P
1 (yearly)	$ 10600.00
2 (semiannually)	$ 10609.00
4 (quarterly)	$ 10613.64

12 (monthly)	$ 10616.78
52 (weekly)	$ 10618.00
365 (daily)	$ 10618.31
continuous	$ 10618.37

Loan Balance

Situation: A person initially borrows an amount *A* and in return agrees to make *n* repayments per year, each of an amount *P*. While the person is repaying the loan, interest is accumulating at an annual percentage rate of *r*, and this interest is compounded n times a year (along with each payment). Therefore, the person must continue paying these installments of amount *P* until the original amount and any accumulated interest is repaid. This equation gives the amount *B* that the person still needs to repay after *t* years.

$$B = A\,(1 + r/n)^{NT} - P\,\frac{(1 + r/n)^{NT} - 1}{(1 + r/n) - 1}$$

where
B = balance after *t* years
A = amount borrowed
n = number of payments per year
P = amount paid per payment
r = annual percentage rate (APR)

52)

Lengths

(**Math** | **General** | **Weights and Measures** | Lengths)

Unit Conversion Tables for Lengths & Distances

A note on the metric system:
Before you use this table, convert to the base measurement first. For example, convert centimeters to meters, convert kilograms to grams.

The notation 1.23**E - 4** stands for $1.23 \times 10^{-4} = 0.000123$.

from \ to	= ___ feet	= __ inches	= __ meters	= __ miles	= __ yards
foot		12	0.3048	(1/5280)	(1/3)
inch	(1/12)		0.0254	(1/63360)	(1/36)
meter	3.280839...	39.37007...		6.213711...**E - 4**	1.093613...
mile	5280	63360	1609.344		1760
yard	3	36	0.9144	(1/1760)	

Young Scientist M.H.Rana, www.matherana.synthasite.com

To use: Find the unit to convert **from** in the left column, and multiply it by the expression under the unit to convert **to**.

Examples: foot = 12 inches; 2 feet = 2x12 inches.

Useful Exact Length Relationships

mile = 1760 yards = 5280 feet
yard = 3 feet = 36 inches
foot = 12 inches
inch = 2.54 centimeters

53)

Metric Prefixes

(Math | General | Weights and Measures | Metric Prefixes)

Metric Prefix Table

Number	Prefix	Symbol	Number	Prefix	Symbol
10^{1}	deka-	da	10^{-1}	deci-	d
10^{2}	hecto-	h	10^{-2}	centi-	c
10^{3}	kilo-	k	10^{-3}	milli-	m
10^{6}	mega-	M	10^{-6}	micro-	µ
10^{9}	giga-	G	10^{-9}	nano-	n
10^{12}	tera-	T	10^{-12}	pico-	p
10^{15}	peta-	P	10^{-15}	femto-	f
10^{18}	exa-	E	10^{-18}	atto-	a
10^{21}	zeta-	Z	10^{-21}	zepto-	z
10^{24}	yotta-	Y	10^{-24}	yocto-	y

Online Unit Converters

54)

Multiplication Table

(Math | General | MultiplicationTable

Multiplication Table

12	0	12	24	36	48	60	72	84	96	108	120	132	144
11	0	11	22	33	44	55	66	77	88	99	110	121	132
10	0	10	20	30	40	50	60	70	80	90	100	110	120
9	0	9	18	27	36	45	54	63	72	81	90	99	108
8	0	8	16	24	32	40	48	56	64	72	80	88	96
7	0	7	14	21	28	35	42	49	56	63	70	77	84
6	0	6	12	18	24	30	36	42	48	54	60	66	72
5	0	5	10	15	20	25	30	35	40	45	50	55	60
4	0	4	8	12	16	20	24	28	32	36	40	44	48
3	0	3	6	9	12	15	18	21	24	27	30	33	36
2	0	2	4	6	8	10	12	14	16	18	20	22	24
1	0	1	2	3	4	5	6	7	8	9	10	11	12
0	0	0	0	0	0	0	0	0	0	0	0	0	0
X	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**

Alternative Format

X	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10	11	12
2	0	2	4	6	8	10	12	14	16	18	20	22	24
3	0	3	6	9	12	15	18	21	24	27	30	33	36
4	0	4	8	12	16	20	24	28	32	36	40	44	48
5	0	5	10	15	20	25	30	35	40	45	50	55	60
6	0	6	12	18	24	30	36	42	48	54	60	66	72
7	0	7	14	21	28	35	42	49	56	63	70	77	84
8	0	8	16	24	32	40	48	56	64	72	80	88	96

9	0	9	18	27	36	45	54	63	72	81	90	99	108
10	0	10	20	30	40	50	60	70	80	90	100	110	120
11	0	11	22	33	44	55	66	77	88	99	110	121	132
12	0	12	24	36	48	60	72	84	96	108	120	132	144

55)

Number Notation

(**Math** | **General** | **Number Notation**)

Names | SI (Metric) Prefixes | Roman Numerals | Bases

Hierarchy of Decimal Numbers

Number	Name	How many
0	zero	
1	one	•
2	two	••
3	three	•••
4	four	••••
5	five	•••••
6	six	••••••
7	seven	•••••••
8	eight	••••••••
9	nine	•••••••••
10	ten	••••••••••
20	twenty	two tens
30	thirty	three tens
40	forty	four tens
50	fifty	five tens
60	sixty	six tens
70	seventy	seven tens
80	eighty	eight tens
90	ninety	nine tens

Number	Name	How Many
100	one hundred	ten tens
1,000	one thousand	ten hundreds
10,000	ten thousand	ten thousands
100,000	one hundred thousand	one hundred thousands
1,000,000	one million	one thousand thousands

Some people use a comma to mark every 3 digits. It just keeps track of the digits and makes the numbers easier to read.

Beyond a million, the names of the numbers differ depending where you live. The places are grouped by thousands in America and France, by the millions in Great Britain and Germany.

Name	American-French	English-German
million	1,000,000	1,000,000
billion	1,000,000,000 (a thousand millions)	1,000,000,000,000 (a million millions)
trillion	1 with 12 zeros	1 with 18 zeros
quadrillion	1 with 15 zeros	1 with 24 zeros
quintillion	1 with 18 zeros	1 with 30 zeros
sextillion	1 with 21 zeros	1 with 36 zeros
septillion	1 with 24 zeros	1 with 42 zeros
octillion	1 with 27 zeros	1 with 48 zeros
googol	1 with 100 zeros	
googolplex	1 with a googol of zeros	

Fractions

Digits to the right of the decimal point represent the fractional part of the decimal number. Each place value has a value that is one tenth the value to the immediate left of it.

Number	Name	Fraction
.1	tenth	1/10
.01	hundredth	1/100
.001	thousandth	1/1000
.0001	ten thousandth	1/10000
.00001	hundred thousandth	1/100000

Examples:

0.234 = 234/1000 (said - point 2 3 4, or 234 thousandths, or two hundred thirty four thousandths)

4.83 = 4 83/100 (said - 4 point 8 3, or 4 and 83 hundredths)

SI Prefixes

Number	Prefix	Symbol	Number	Prefix	Symbol
10^{1}	deka-	da	10^{-1}	deci-	d
10^{2}	hecto-	h	10^{-2}	centi-	c
10^{3}	kilo-	k	10^{-3}	milli-	m
10^{6}	mega-	M	10^{-6}	micro-	u (greek mu)
10^{9}	giga-	G	10^{-9}	nano-	n
10^{12}	tera-	T	10^{-12}	pico-	p
10^{15}	peta-	P	10^{-15}	femto-	f
10^{18}	exa-	E	10^{-18}	atto-	a
10^{21}	zeta-	Z	10^{-21}	zepto-	z
10^{24}	yotta-	Y	10^{-24}	yocto-	y

Roman Numerals

I=1		(I with a bar is not used)
V=5		$\overline{V}$=5,000
X=10		$\overline{X}$=10,000
L=50		$\overline{L}$=50,000
C=100		$\overline{C}$ = 100 000
D=500		$\overline{D}$=500,000
M=1,000		$\overline{M}$=1,000,000

Roman Numeral Calculator

Examples:

1 = I	11 = XI	25 = XXV
2 = II	12 = XII	30 = XXX

3 = III	13 = XIII	40 = XL
4 = IV	14 = XIV	49 = XLIX
5 = V	15 = XV	50 = L
6 = VI	16 = XVI	51 = LI
7 = VII	17 = XVII	60 = LX
8 = VIII	18 = XVIII	70 = LXX
9 = IX	19 = XIX	80 = LXXX
10 = X	20 = XX	90 = XC
	21 = XXI	99 = XCIX

There is no zero in the roman numeral system.

The numbers are built starting from the largest number on the left, and adding smaller numbers to the right. All the numerals are then added together.

The exception is the subtracted numerals, if a numeral is before a larger numeral, you subtract the first numeral from the second. That is, IX is 10 - 1= 9.

This only works for one small numeral before one larger numeral - for example, IIX is not 8, it is not a recognized roman numeral.

There is no place value in this system - the number III is 3, not 111.

Number Base Systems

Decimal(10)	Binary(2)	Ternary(3)	Octal(8)	Hexadecimal(16)
0	0	0	0	0
1	1	1	1	1
2	10	2	2	2
3	11	10	3	3
4	100	11	4	4
5	101	12	5	5
6	110	20	6	6
7	111	21	7	7
8	1000	22	10	8
9	1001	100	11	9
10	1010	101	12	A
11	1011	102	13	B

12	1100	110	14	C
13	1101	111	15	D
14	1110	112	16	E
15	1111	120	17	F
16	10000	121	20	10
17	10001	122	21	11
18	10010	200	22	12
19	10011	201	23	13
20	10100	202	24	14

Each digit can only count up to the value of one less than the base. In hexadecimal, the letters A - F are used to represent the digits 10 - 15, so they would only use one character.

56)

PI

(**Math** | **Miscellaneous** | **Constants** | PI)

Pi is a name given to the ratio of the circumference of a circle to the diameter. That means, for any circle, you can divide the circumference (the distance around the circle) by the diameter and always get exactly the same number. It doesn't matter how big or small the circle is, Pi remains the same. Pi is often written using the symbol $\pi$and is pronounced "pie", just like the dessert.

History | Pi web sites| Do it yourself Pi |The Digits| Formulas

A Brief History of Pi
Ancient civilizations knew that there was a fixed ratio of circumference to diameter that was approximately equal to three. The Greeks refined the process and Archimedes is credited with the first theoretical calculation of Pi.

In 1761 Lambert proved that Pi was irrational, that is, that it can't be written as a ratio of integer numbers.

In 1882 Lindeman proved that Pi was transcendental, that is, that Pi is not the root of any algebraic equation with rational coefficients. This discovery proved that you can't "square a circle", which was a problem that occupied many mathematicians up to that time. (More information on squaring the circle.)

How many digits are there? Does it ever end?
Because Pi is known to be an irrational number it means that the digits never end or repeat in any known way. But calculating the digits of Pi has proven to be an fascination for mathematicians throughout history. Some spent their lives calculating the digits of Pi, but until computers, less than 1,000 digits had been calculated. In 1949, a computer calculated 2,000 digits and the race was on. Millions of digits have been calculated, with the record held (as of September 1999) by a supercomputer at the University of Tokyo that calculated 206,158,430,000 digits. (first 1,000 digits)

More about the History of Pi can be found at the Mac Tutor Math History archives.

Approximation of Pi

Archimedes calculated that Pi was between **3 *10/71 and 3 1/7*** (also written **223/71** < π < **22/7**). **22/7** is still a good approximation. **355/113** is a better one.

Pi Web Sites

Pi continues to be a fascination of many people around the world. If you are interested in learning more, there are many web sites devoted to the number Pi. There are sites that offer thousands, millions, or billions of digits, pi clubs, pi music, people who calculate digits, people who memorize digits, Pi experiments and more. Check this Yahoo page for a complete listing.

A Cool Pi Experiment

One of the most interesting ways to learn more about Pi is to do pi experiments yourself. Here is a famous one called **Buffon's Needle.**

In Buffon's Needle experiment you can drop a needle on a lined sheet of paper. If you keep track of how many times the needle lands on a line, it turns out to be directly related to the value of Pi.

Buffon's Needle Simulation Applet (Michael J. Hurben)
Buffon's Needle (George Reese, Office for Mathematics, Science and Technology Education University of Illinois Champaign-Urbana)

Digits of Pi

First 100 digits

3.1415926535 8979323846 2643383279 5028841971 6939937510 5820974944 5923078164 0628620899 8628034825 3421170679 ...

First 1000 digits

3.1415926535 8979323846 2643383279 5028841971 6939937510 5820974944 5923078164
0628620899 8628034825 3421170679 8214808651 3282306647 0938446095 5058223172 5359408128
4811174502 8410270193 8521105559 6446229489 5493038196 4428810975 6659334461 2847564823
3786783165 2712019091 4564856692 3460348610 4543266482 1339360726 0249141273 7245870066
0631558817 4881520920 9628292540 9171536436 7892590360 0113305305 4882046652 1384146951
9415116094 3305727036 5759591953 0921861173 8193261179 3105118548 0744623799 6274956735
1885752724 8912279381 8301194912 9833673362 4406566430 8602139494 6395224737 1907021798
6094370277 0539217176 2931767523 8467481846 7669405132 0005681271 4526356082 7785771342
7577896091 7363717872 1468440901 2249534301 4654958537 1050792279 6892589235 4201995611
2129021960 8640344181 5981362977 4771309960 5187072113 4999999837 2978049951 0597317328
1609631859 5024459455 3469083026 4252230825 3344685035 2619311881 7101000313 7838752886
5875332083 8142061717 7669147303 5982534904 2875546873 1159562863 8823537875 9375195778
1857780532 1712268066 1300192787 6611195909 2164201989

5 million, 10 million, 100 million, and 200 million digits

Formulas for Pi

More complex formulas and derivations

Vieta's Formula

$2/PI = \sqrt{2}/2 * \sqrt{(2 + \sqrt{2})}/2 * \sqrt{(2 + (\sqrt{(2 + \sqrt{2}) }))}/2 * \ldots$c

Leibnitz's Formula

$PI/4 = 1/1 - 1/3 + 1/5 - 1/7 + \ldots$

Wallis Product

$PI/2 = 2/1 * 2/3 * 4/3 * 4/5 * 6/5 * 6/7 * \ldots$

$2/PI = (1 - 1/2^2)(1 - 1/4^2)(1 - 1/6^2)\ldots$

Lord Brouncker's Formula

$$4/PI = 1 + \cfrac{1}{2 + \cfrac{3^2}{2 + \cfrac{5^2}{2 + 7^2 \ldots}}}$$

$$\mathbf{(PI^2)/8 = 1/1^2 + 1/3^2 + 1/5^2 + \ldots}$$

$$\mathbf{(PI^2)/24 = 1/2^2 + 1/4^2 + 1/6^2 + \ldots}$$

Euler's Formula

$$(PI^2)/6 = \sum(n = 1..\infty)\ 1/n^2 = 1/1^2 + 1/2^2 + 1/3^2 + \ldots$$

(or more generally...)

$$\sum(n = 1..\infty)\ 1/n^{(2k)} = (-1)^{(k-1)}\ PI^{(2k)}\ 2^{(2k)}\ B_{(2k)}\ /\ (\ 2(2k)!)$$

$B_{(k)}$ = the k th Bernoulli number. eg. $B_0=1$ $B_1=-1/2$ $B_2=1/6$ $B_4=-1/30$ $B_6=1/42$ $B_8=-1/30$ $B_{10}=5/66$. Further Bernoulli numbers are defined as $(n\ 0)B_0 + (n\ 1)B_1 + (n\ 2)B_2 + \ldots + (n\ (n-1))B_{(N-1)} = 0$ assuming all odd Bernoulli #'s > 1 are = 0. (n k) = binomial coefficient = n!/(k!(n-k)!)

See Power Summations #2 for simplified expressions (without the Bernoulli notation) of these sums for given values of k.

57) Technical Support

(1) Yarn Count Definitions

Metric Count (Nm): **Nm** = m / 1-g	**Denier Count (den):** **Den** = g / 9000-m
English Cotton Count (Ne_B): **Ne_B** = 840-yd / 1-lb *or* **Ne_B** = 768.1-m / 453.59-g	**Tex Count (tex):** **Tex** = g / 1000-m

(2) Yarn Count Conversion Factors

From metric count (Nm) to others:
Tex = 1000 / Nm
Ne_B = 0.59 x Nm
Den = 9000 / Nm
From denier (den) to others:
Nm = 9000 / den
Ne_B = 5315 / den
Tex = 0.111 x den
From English cotton count (Ne_B) to others:
Nm = 1.693 x Ne_B
Tex = 590 / Ne_B
Den = 5314 / Ne_B
From tex count (tex) to others:
Nm = 1000 / tex
Ne_B = 590 / tex
Den = 9 x tex

Conversion examples:
[Ex. 1: Ne_B 30 = 5315/30 den = 177 den] [Ex. 2: 150 den = Ne_B 5315 / 150 = Ne_B 35] [Ex. 3: Ne_B 20 = Nm 1.693 x 20 = Nm 34]

58)

Volumes

(Math | General | Weights & Measures | Volumes)

Unit Conversion Tables for Volumes

A note on the metric system:

Before you use this table, convert to the base measurement first. For example, convert centimeters to meters, kilograms to grams, etc.

The notation 1.23**E - 4** stands for 1.23 x 10^{-4} = 0.000123.

from \ to	= __ **feet^3**	= __ **gallons**	= __ **inches^3**	= __ **liters**	= __ **meters^3**	= __ **miles^3**	= __ **pints**	= __ **quarts**	= __ **yards^3**
foot^3		7.480519...	1728	28.31684...	0.02831684...	6.793572**E - 12**	59.84415...	29.92207...	(1/27)
gallon	0.1336805...		231	3.785411...	0.003785411...	9.081685... **E - 13**	8	4	0.004951131...
inch^3	(1/1728)	(1/231)		0.01638706...	1.638706...**E - 5**	3.931465... **E - 15**	(1/28.875)	(1/57.75)	(1/46656)
liter	0.03531466...	0.2641720...	61.02374...		(1/1000)	2.399127... **E - 13**	2.113376...	1.056688...	0.001307950...
meter^3	35.31466...	264.1720...	61023.74...	1000		2.399127... **E - 10**	2113.376...	1056.688...	1.307950...
mile^3	1.471979...**E + 11**	1.101117...**E + 12**	2.543580 **E + 14**	4.168181...**E + 12**	4.168181...**E + 9**		8.808937...**E + 12**	4.404468... **E + 12**	5.451776... **E + 9**
pint	0.01671006...	(1/8)	28.875	0.4731764...	4.731764...**E - 4**	1.135210... **E - 13**		(1/2)	6.188914... **E - 4**
quart	0.03342013...	(1/4)	57.75	0.94635...	9.463529...**E - 4**	2.270421... **E - 13**	2		0.001237782...
yard^3	27	201.974...	46656	764.555...	0.7645548...	1.834264... **E - 10**	1615.792...	807.8961...	

To use: find the unit to convert **from** in the left column, and multiply it by the expression under the unit to convert **to**.
Examples: foot^3 = 1728 inches^3; 2 feet^3 = 2x1728 inches^2.

Useful Exact Volume Relationships

fluid ounce = (1/8) cup = (1/16) pint = (1/32) quart = (1/128) gallon
gallon = 128 fluid ounces = **231 inches^3** = 8 pints = 4 quarts
quart = 32 fluid ounces = 4 cups = 2 pints = (1/4) gallon

Useful Exact Length Relationships

cup = 8 fluid ounces = (1/2) pint = (1/4) quart = (1/16) gallon
mile = 63360 inches = 5280 feet = 1760 yards
yard = 36 inches = 3 feet = (1/1760) mile
foot = 12 inches = (1/3) yard = (1/5280) mile
pint = 16 fluid ounces = (1/2) quart = (1/8) gallon

inch = **2.54 centimeters** = (1/12) foot = (1/36) yard
liter = 1000 centimeters^3 = **1** $\textbf{decimeter}^3$ = (1/1000) meter^3
Note that when converting volume units:
1 foot = 12 inches
$(1 \text{ foot})^3 = (12 \text{ inches})^3$ (cube both sides)
1 $\textbf{foot}^3$ **= 1728** $\textbf{inches}^3$
The linear & volume relationships are not the same!

Online Unit Converters

59)

Math Tables: *Weights and Measures*

(**Math** | **General** | **Weights and Measures** | **Areas**)

Unit Conversion Tables for Areas

A note on the metric system:
Before you use this table **convert to the base measurement first.** For example, convert **centimeters to meters**, convert **kilograms to grams**.

The notation 1.23**E - 4** stands for $1.23 \times 10^{-4} = 0.000123$.

from \ to	= __ acres	= __ feet^2	= __ inches^2	= __ meters^2	= __ miles^2	= __ yards^2
acre		43560	6272640	4046.856...	(1/640)	4840
$\textbf{foot}^2$	(1/43560)		144	0.09290304	(1/27878400)	(1/9)
$\textbf{inch}^2$	(1/6272640)	(1/144)		6.4516**E - 4**	2.490977**E - 10**	(1/1296)
$\textbf{meter}^2$	2.471054...**E - 4**	10.76391...	1550.0031		3.861021...**E - 7**	1.195990...
$\textbf{mile}^2$	640	27878400	4.0145**E + 9**	2.589988...**E + 6**		3097600

yard^2	(1/4840)	9	1296	0.83612736	3.228305...**E - 7**	

To use: Find the unit to convert **from** in the left column, and multiply it by the expression under the unit to convert **to**.
Examples: foot^2 = <u>144</u> inches^2; 2 feet^2 = <u>2x144</u> inches^2.

Useful Exact Area & Length Relationships

acre = (1/640) miles^2
mile = 1760 yards = 5280 feet
yard = 3 feet = 36 inches
foot = 12 inches
inch = 2.54 centimeters

Note that when converting area units:
1 foot = 12 inches
(1 foot)^2 = (12 inches)^2 (square both sides)
1 foot^2 = 144 inches^2
The linear & area relationships are not the same!

60) Unit equivalent:

1 Centimeters = 1 Centimeters 1 Inches = 2.54000 Centimeters 1 Feet = 30.48000 Centimeters 1 Yards = 91.44000 Centimeters 1 Meters = 100 Centimeters 1 Chains = 2012 Centimeters 1 Kilometers = 100000 Centimeters 1 Miles = 160934 Centimeters	1 Centimeters = 0.39370 Inches 1 Feet = 12 Inches 1 Yards = 36 Inches 1 Meters = 39.37000 Inches 1 Chains = 792 Inches 1 Kilometers = 39370 Inches 1 Miles = 63360 Inches
1 Centimeters = 0.03281 Feet 1 Inches = 0.08333 Feet	1 Centimeters = 0.01094 Yards 1 Inches = 0.02778 Yards

1 Yards = 3 Feet 1 Meters = 3.28100 Feet 1 Chains = 66 Feet 1 Kilometers = 3281 Feet 1 Miles = 5280 Feet	1 Feet = 0.33330 Yards 1 Meters = 1.09360 Yards 1 Chains = 22 Yards 1 Kilometers = 1093.60000 Yards 1 Miles = 1760 Yards
1 Centimeters = 0.01000 Meters 1 Inches = 0.02540 Meters 1 Feet = 0.30480 Meters 1 Yards = 0.91440 Meters 1 Chains = 20.12000 Meters 1 Kilometers = 1000 Meters 1 Miles = 1609 Meters	1 Centimeters = 0.00049 Chains 1 Inches = 0.00126 Chains 1 Feet = 0.01515 Chains 1 Yards = 0.04545 Chains 1 Meters = 0.04971 Chains 1 Kilometers = 49.71000 Chains 1 Miles = 80 Chains
1 Centimeters = 0.00001 Kilometers 1 Inches = 0.00002 Kilometers 1 Feet = 0.00030 Kilometers 1 Yards = 0.00091 Kilometers 1 Meters = 0.00100 Kilometers 1 Chains = 0.02120 Kilometers 1 Miles = 1.60900 Kilometers	1 Centimeters = 0.00000 Miles 1 Inches = 0.00001 Miles 1 Feet = 0.00019 Miles 1 Yards = 0.00056 Miles 1 Meters = 0.00062 Miles 1 Chains = 0.01250 Miles 1 Kilometers = 0.62140 Miles

Spinning Calculations

Count:-

Count is the measure of fineness or coarseness of yarn.

Systems of Count Measurement

There are two systems for the measurement of count.

1) Direct System
2) Indirect System

1) Direct System

It is used for the measurement of weight per unit length of yarn.
When count increases, fineness decreases. (count↑ fineness↓)
Commonly used units in this system of measurement are:-

1) Tex (1 Tex = 1g/ 1000m)
2) Grex (1 Grex = 1g/ 10,000m)
3) Denier (1 Denier = 1g/ 9000m)

2) Indirect System:-

It is used for the measurement of length per unit weight of yarn.

When count increases, fineness increases. (count↑ fineness↑)
Commonly used subsystems of indirect system are:-

1) English System (1 Ne = 1 Hank/ lb)
2) Metric System (1 Nm = 1 Km/ kg)

For cotton yarn, length of 1 Hank = 840 yards.
Whenever the type of count is not mentioned with the count, it is understood that it is the English count.

Basic Conversions

No	Length	Weight	Time
1.	1 in = 2.54 cm	1 lb = 7000 gr	1 min = 60 sec
2.	1 yd = 36 in	1 lb = 16 oz	1 hr = 60 min
3.	1 m = 1.0936 yd	1 oz = 437.5 gr	1 shift = 8 hr
4.	1 Hk = 840 yd	1 kg = 2.2046 lb	1 day = 24 hr
5.	1 Hk = 7 leas	1 bag = 100 lb	1 day = 3 shifts

Abbreviations:
In [inch(es)], yd [yard(s)], kg [kilogram(s)], m [meter(s)], Hk [hank(s)], lb [pound(s)], oz [ounce(s)], gr [grain(s)] , sec [second(s)], min [minute(s)], [hour(s)].

Count Conversion Table

	Ne	Nm	Tex	Grex	Denier
Ne=	1 xNe	0.5905 xNm	590.5 /Tex	5905 /Grex	5315 /Den
Nm=	1.693xNe	1 xNm	1000 /Tex	10,000/Grex	9000 /Den
Tex=	590.5 /Ne	1000 /Nm	1 xTex	0.1 xGrex	0.111 xDen
Grex=	5905 /Ne	10,000 /Nm	10 xTex	1 xGrex	1.111 xDen
Denier=	5315 /Ne	9000 /Nm	9 xTex	0.9 xGrex	1 xDen

Derivation:-

Ne = 0.5905 Nm

Let us suppose we have,
Total Ne = **x** Ne
Ne = **x** Hanks/ lb

This means that,
x Hanks are in ------------------ 1 lb

840x yards in ------------------ 1 lb

840 x m in ------------------- 1 lb

1.0936

840 x x 2.2046 m in---------- 1 lb

1.0936 x 1000

(We know that, Nm= km/ kg = m/ g.

840 x x 2.2046 m/ g Since this value has the units of Nm

1.0936 x 1000 so it equals Nm.)

Nm = 840 x 2.2046 x

1.0936 x 1000

Nm = 1.693 x (as x = Ne,)

Nm = 1.693 Ne

Ne = 0.5905 Nm

Yarn Classification

(on the basis of no. of plies)

1) Single yarn

e.g 80/1 (read as 80 single) means 80 fibres twisted to form a single yarn.

2) Plied yarn

e.g. 80/2 (read as 80 double) means 80 fibres twisted to form two individual yarns.

The number of plied yarns may exceed two.

Draft & TPI Formulas

Surface speed = DN / min

D = dia. of rotating element

N = rpm (no. of revolutions/min)

Mechanical Draft = S.S of Front roller (D N) > 1

S.S of Back roller (D N)

Actual Draft = count delivered

count fed

Indirect system

A. D. = 1 / w delivered

l / w fed
Actual Draft = count fed
count delivered
Direct system
A.D. = w / l fed .
w / l delivered

No. of Twists Per Inch, TPI = rpm of flyer simplex
S.S of F.R

Numerical Problems

1) Calculate the length of a package of 80/1 and cone weight 2.083 lb.
(Note:- English count is represented as C/N i-e, yarn count/ no. of yarn plies)
Yarn type = 80/1
Cone wt. = 2.083 lb
Cone length = ?

Solution:-
length = Ne x lb x 840 yards
= 80 x 2.083 x 840 yards
= 139977.6 m
1.0936
= 127997.07 m ------------Ans.

2) Calculate the length of yarn with Ne (80/2) and weight 4.166 lb :-
Yarn type = 80/2
Cone weight = 4.166 lb
Cone length = ?

Solution:-
length = Ne x lb x 840 yards
= 80 x 4.166 x 840 yards
2

= 139977.6 m
1.0936
= 127997.07 m ---------------Ans.

3) Calculate the draft at drawing frame if the feeding sliver is 68 grains/ yard, delivered sliver is 48 grains/ yard and the number of doublings is 8 :-

Count of feeding sliver = 68 gr/ yd
Count of delivered sliver = 48 gr/ yd
Doubling = 8 (8 sliver cans used)
Draft = ?

Solution:-
Actual draft = count fed x doubling (direct system)
count delivered
= 68 x 8
48
= 11.33-------------Ans.

4) Calculate the grains/ yard of delivered sliver if feeding sliver is 68, doubling is 6 and the draft is 7 :-

Count of F.S = 68
Count of D.S = ?
Doubling = 6
Draft = 7

Solution :-
A.D = F.S x D
D.S
7 = 68 x 6
D.S
D.S = 68 x 6 = 58.28 grains/ yard ----------Ans.
7

5) Calculate the draft if feeding sliver is 60 gr/ yd, delivered sliver is 1 HS and doubling is 6 :-

Count of F.S = 60 gr/yd

Count of D.S = 1 HS

Doubling, D = 6

Draft = ?

Solution :-

60 gr in-------------------- 1 yd

60 lb in-------------------- 1 yd

7000

60 x 840 lb in----------- 840 yd

7000

60 x 840 lb/ Hank (direct count)

7000

7000 x 1 Hank/ lb (indirect count)

60 840

= 0.139 Hank/ lb

= 0.139 Ne

Actual Draft = count del. = 1 . = 43.6 ----------Ans.

count fed 0.139/ 6

6) Calculate the English count of delivered sliver on drawing frame when doubling is 6, count of feeding sliver is 70 gr/ yd, diameter of front roller is 30 mm and its rpm is 100, whereas the diameter of back roller is 15 mm and its rpm is 10 :-

Count of D.S = ?

Count of F.S = 70 gr/ yd

Doubling, D = 6

Dia. of F.R, DF = 30 mm

Dia. of B.R, DB = 15 mm

Rpm of F.R, NF = 100

Rpm of B.R, NB = 10

Solution :-

F.S. = 70 gr/ yd = 70 grains in--------------- 1 yd

= 70 lb in------------------ 1 yd

7000

= 0.01 lb in------------------ 1 yd

= (0.01 x 840) lb in---------- 840 yd

= 8.4 lb in--------------------- 840 yd

= 8.4 lb in--------------------- 1 Hank

= 1/ 8.4 Hanks/ lb

= 0.119 Hanks/ lb = 0.119 H.S (Ne).

Mechanical Draft = S.S of F.R = $\pi D_F N_F$ = 30 x 100 = 20

S.S of B.R $\pi D_B N_B$ 15 x 10

On drawing frame, neither twist is inserted nor the waste is produced so we have;

Mechanical draft = Actual draft = 20

Now in case of indirect count, A.D = count delivered

count fed

A.D = D.S .

F.S/ D

20 = D.S .

0.119 / 6

D.S = 20 x 0.119

6

= 0.396 H.S (Ne)---------------Ans.

7) Calculate the TPI (twists per inch) produced on a simplex with diameter of front roller 28 mm and its rpm be 30. The rpm of flyer is 1000.

TPI on simplex = ?

Dia. of F.R = 28 mm = 2.8 cm

Rpm of F.R = 30

Rpm of flyer = 1000

Solution :-
Dia. of Front roller = 2.8 cm / 2.54 (1 in= 2.54 cm)
= 1.1023 inch
Surface speed of F.R, ˆDN = π x dia. of F.R x rpm of F.R
= π x 1.1023 x 30
= 103.88 “/ min.
TPI = rpm of flyer = 1000 = 9.63 -----------------Ans.
S.S of F.R 103.88

8) Calculate the TPI on simplex if the diameter of back roller is 15/16” , rpm of B.R is 10, rpm of flyer is 1000 and draft is 6 :-
TPI on simplex = ?
Dia. of B.R = 15/16”
Dia. of F.R = ?
Rpm of B.R = 10
Rpm of flyer = 1000 rpm
Draft, D = 6

Solution :-
S.S of B.R = πDN = π x 15/16” x 10 = 29.45” / min
D = S.S of F.R => 6 = S.S of F.R
S.S of B.R 29.45
S.S of F.R = 6 x 29.45 = 176.71” / min
TPI = rpm of flyer = 1000 . = 5.66 -----------------Ans.
S.S of F.R 176.71

Production calculations

☐ Production

The output of a m/c per unit time is called its production. The production is usually calculated in the units of weight/time or length/time e.g, oz/hr, lb/shift,
yd/hr, Hk/day etc.
The most commonly used unit of time for production calculation is hour. So if

the unit is not mentioned, it is understood to be a production/hr.

☐ Efficiency

It is the ability of a material to perform its task.
In other words, it is the ratio of the output of a m/c to the input of that m/c.

Mathematically,
Efficiency = output
Input
Its value ranges from 0→1. it has no units.

☐ Efficiency Percentage

It is the %age performance of a m/c.

Mathematically,
Efficiency = output x 100
Input
Its value ranges from 0→100.
If the efficiency of a m/c is 0.8, its percentage efficiency 80. The word ‘percent’ means ‘per 100’ which suggests that the efficiency is 80 .
100

☐ Cleaning Efficiency (%)

It is the ratio of the trash extracted to the total trash content in a material.
For any m/c, mathematically,
Cleaning eff. = trash in fed material - trash in del. material
trash in fed material

☐ Beating action

The regular hard hits or strikes made by a rotating beater through a material (for its opening or cleaning) are known as beating action.

☐ Beats per inch

The no. of beats made by a beater per inch of a material surface is known as beating action.
Mathematically,
Beats/inch = beater rpm x no. of arms
π x feed roller dia" x feed roller rpm

☐ Twists per inch

Twist insertion & draft in a sliver gives roving and further twisting and drafting of roving gives yarn.
So the no. of twists in one inch of yarn (or roving) is known as TPI (twists perinch).

Mathematically,
TPI = spindle speed (rpm) .
Front roller delivery (in/min)
Also,
TPI $\sqrt{\text{count}}$
TPI = TM x $\sqrt{\text{count}}$

Hank:

The word 'Hank' is used in two ways. Literally, it is a unit of length, i-e;
1 Hank = 840 yard but practically, we take it as a unit of English count, i-e;
1 Hank = 840 yd/lb
2 Hank = 1680 yd/lb

Roller Speeds:

In spinning calculations, we deal in two kinds of roller speeds, i-e; surface speed and rotating speed (rpm). So when the speed of a roller is mentioned without any units, this means that it is the rpm of the roller, e-g;
speed = 20 means
speed = 20 rpm

PRODUCTION FORMULAS

1. Production of Scutcher

P = π DN x 60 x lap ct. (oz/yd) x η [oz/hr]36

2. Production of Card m/c

P = π DN x 60 x sliver ct. (gr/yd) x η x tension draft [lb/hr]36 7000

3. Production of Draw frame

P = π DN x 60 x del. sliver ct. (gr/yd) x η x no. of x no. of [lb/hr]
36 7000 heads m/c

4. Production of Lap Former

P = π DN x 60 x lap ct. (gr/yd) x η x no. of m/c [lb/hr]36 7000

5. Production of Comber

P = f (π DN) x 60 x sliver ct. (gr/yd) x η x N x no. of x no. of x 1 - w .
36 7000 heads m/c 100 [lb/hr]

6. Production of Simplex

P = π DN x 60 x roving ct. (gr/yd) x η x no. of spindles [lb/hr] 36 7000

7. Production of Ring frame

P = π DN x 60 x 16 x 8 x η [oz/shift/spindle]
TPI x 36 840 x ct.

P = P [oz/shift/spindle] x no. of spindles x no. of frames [oz/shift]

DETAILS OF PRODUCTION FORMULAS

DERIVATIONS & PROBLEMS

1.Production of Scutcher

P = π DN x 60 x lap ct.(oz/yd) x η [oz/hr]
36

DERIVATION:

Let D = dia. of lap roller (in inches)
N = rpm of lap roller
η = efficiency of m/c
P = production

Production = surface speed of lap roller x lap ct. (wt/l)
= πDN (in/min) x lap (oz/yd)
= π DN (yd/min) x lap (oz/yd)36
= π DN x 60 (yd/hr) x lap (oz/yd)36

Since the efficiency of a m/c is always less than 1 so,
= π DN x 60 (yd/hr) x lap (oz/yd) x η36
= π DN x 60 (yd/hr) x lap (oz/yd) x η [oz/hr]36

The value (πDN/36) x 60 may be taken as a production constant when working on a m/c with a fixed dia. and rpm of delivery roller.
The delivery speed of a pair of rollers is the same as its surface speed. So the value π DN can also be mentioned as delivery speed.

P = P [oz/hr] [lb/hr]16
P = P [lb/hr] x 8 [lb/shift]
P = P [lb/hr] x 24 [lb/day]
P = P [oz/hr] [kg/hr]16 x 2.2046

Also,
P = π DN x 60 x 1 x η [lb/hr]36 840 Ne
but let us not use this formula to avoid confusions.

Q:1-Calculate the production of scutcher if the lap wt. is 13 oz/yd, and the dia and speed of shell roller are 11 rpm and 240 mm respectively.

Furnish the production in lb/hr, kg/hr, lb/shift, kg/shift and bag/day when the efficiency of the m/c is 75%:-

Lap wt/l = 13 oz/yd

Shell roller speed, N = 11 rpm

Shell roller dia., D = 240 mm = 9.5"

Efficiency, η = 75% = 75/100 = 0.75

P [lb/hr], P [kg/hr] , P [lb/shift], P [kg/shift] & P[bag/day] = ?

Solution:-

P [oz/hr] = π DN x 60 x lap ct. (oz/yd) x η [oz/hr]36

= π x 9.5 x 11 x 60 x 13 (oz/yd) x 0.75 [oz/hr]36

= 9.12 x 60 x 13 (oz/yd) x 0.75 [oz/hr]

= 5334.82 [oz/hr]

P [lb/hr] = P [oz/hr] = 5334.82 = 333.43 [lb/hr] -----Ans 16 16

P [kg/hr] = P [lb/hr] = 333.43 = 151.24 [kg/hr] -----Ans 2.2046 2.2046

P [lb/shift] = P [lb/hr] x 8 = 333.43 x 8 = 2667.44 [lb/shift] -----Ans

P [kg/shift] = P [kg/hr] x 8 = 151.24 x 8 = 1209.92 [kg/shift] -----Ans

P [bag/day] = P [lb/hr] x 24 = 333.43 x 24 = 80.02 [bag/day] 100 100 Ans

Q:2-The fluted lap roller of a scutcher of 9" dia. makes 10 revolutions per minute. If the lap count is 0.00136 Hk, calculate the production of scutcher in one shift at 80% efficiency:-

Lap count = 0.00136 Hk

= 0.00136 x 840 (yd/lb)

= 1 (lb/yd) x 16 = 14 (oz/yd)

0.00136 x 840

Lap roller speed, N = 10 rpm

Lap roller dia., D = 9"

Efficiency, η = 80/100 = 0.8

P [lb/shift] = ?

Solution:-

P [oz/hr] = π DN x 60 x lap ct. (oz/yd) x η [oz/hr]

36

= π x 9” x 10 x 60 x 14 x 0.8 [oz/hr]
36
= 5277.88 [oz/hr]
P [lb/shift] = P [oz/hr] x 8 = 2639 [lb/shift] -------Ans
16

2. Production of Card m/c

P = π DN x 60 x sliver ct.(gr/yd) x η x tension draft [lb/hr]
36 7000

DERIVATION:

Let D = dia. of coiler calendar rollers (in inches)
N = rpm of coiler calendar rollers
η = efficiency of m/c
Production = surface speed of doffer x carded sliver ct. (wt/l)

As the sliver has a lesser wt/l than a lap it is easier to observe its gr/yd rather than its lb/yd.

= πDN (in/min) x sliver (gr/yd)
= π DN (yd/min) x sliver (gr/yd)36
= π DN x 60 (yd/hr) x sliver (gr/yd)36

Since the efficiency of a m/c is always less than 1 and 1lb = 7000 gr so,

= π DN x 60 (yd/hr) x sliver (gr/yd) (lb/yd) x η [lb/hr]36 7000
P [lb/hr] = π DN x 60 x sliver ct.(gr/yd) x η [lb/hr]36 7000

Although mainly dispersion drafting takes place on card m/c but there is a very small tension draft b/w calendar rollers and coiler calendar rollers. Theoretically, this is ignored but is included in mathematical calculations.

In a case when the dia. and speed (rpm) of coiler calendar rollers are given instead of doffer or calendar rollers, then the tension draft is already included in those values and we need not include that in our formula. So,

P [lb/hr] = π D’ N’ x 60 x sliver ct.(gr/yd) x η x tension draft [lb/hr]
36 7000

Here D’ & N’ are assumed to be the dia. & rpm (respectively) of doffer or calendar rollers.

Q:3-What will be the production of a carding engine in 8 hours at 84% efficiency and 5% waste, if the speed of 2" coiler calendar rollers is 125 rpm with the carded sliver weighing 58 gr/yd ?

Carded sliver wt/l = 58 gr/yd

Coiler calendar rollers speed, N = 125 rpm

Coiler calendar rollers dia., D = 2"

Efficiency, η = 84% = 0.84

Waste %age = 5%

P [lb/shift] = ?

Solution:-

P [lb/hr] = π D' N' x 60 x sliver ct.(gr/yd) x η [lb/hr] 36 7000

= π x 2" x 125 x 60 x 58(gr/yd) x 0.84 [lb/hr] 36 7000

= 9.11 [lb/hr]

P [lb/shift] = P [lb/hr] x 8 = 72.9 [lb/shift] -------Ans

Here the waste percentage is not concerned as the given count is of carded (cleaned) sliver and not of lap. Hence it was just a value given to create confusion.

3. Production of Draw frame

P = π DN x 60 x del. sliver ct.(gr/yd) x η x no. of heads [lb/hr]
36 7000

DERIVATION:

Let D = dia. of calendar rollers (in inches)

N = rpm of calendar rollers

η = efficiency of m/c

Production = surface speed of calendar rollers x drawn sliver ct. (wt/l)

= πDN (in/min) x sliver (gr/yd)

= π DN (yd/min) x sliver (gr/yd)36

= π DN x 60 (yd/hr) x sliver (gr/yd)36

= π DN x 60 (yd/hr) x sliver (gr/yd) (lb/yd) x η [lb/hr]36 7000

= π DN x 60 x sliver ct.(gr/yd) x η [lb/hr]36 7000

Since a drawing frame may have more than one delivery ends or heads and also we may use one or more m/cs at a time for drawing the same

kinds of slivers, so to calculate the total production,

P = π DN x 60 x del. sliver ct.(gr/yd) x η x no. of x no. of [lb/hr]
36 7000 heads m/c

Q:4-The 3” diameter calendar rollers of a 6 delivery drawing frame revolves 125 rpm. Calculate the production in pounds if the drawn sliver is 60 gr/yd and the m/c works for 8 hrs at 70% efficiency:-

Drawn sliver wt/l = 60 gr/yd
Calendar rollers speed, N = 125 rpm
Calendar rollers dia., D = 3”
Efficiency, η = 70% = 0.7
P [lb/shift] = ?

Solution:-

P = π DN x 60 x del. sliver ct.(gr/yd) x η x no. of x no. of [lb/hr]
36 7000 heads m/c
= π x 3” x 125 x 60 x 60 (gr/yd) x 0.7 x 6 x 1 [lb/hr] 36 7000
= 32.725 x 2.16 [lb/hr]
= 70.69 [lb/hr]
P [lb/shift] = P [lb/hr] x 8 = 565.52 [lb/shift] -------Ans

4. Production of Lap Former

P = π DN x 60 x lap ct.(gr/yd) x η x no. of m/c [lb/hr] 36 7000

1—Sliver Lap M/c

Q:5-The speed and dia. of the fluted lap drum of a sliver lap m/c are 30 rpm and 16” respectively. If 24 card cans having 0.15 Hk sliver are fed to the m/c, what will be the production in one shift at 70% efficiency?

Feeding sliver count = 0.15 Hk
Lap roller speed, N = 30 rpm
Lap roller dia., D = 16”
Efficiency, η = 70% = 0.7
P [lb/shift] = ?

Solution:-

No. of yards of each sliver delivered in 1 shift at 70% efficiency

= π DN x 60 x 8 hr x η [yd/shift]36

= π x 16” x 30 x 60 x 8 hr x 0.7 [yd/shift] 36

= 14074.34 [yd/shift]

No. of pounds of each sliver delivered in 1 shift at 70% efficiency

= [yd/shift] = 14074.34 yd x lb = 111.7 [lb/shift]

sliver count shift 0.15 x 840 yd

No. of pounds/yard of each sliver delivered

= [lb/shift] = 111.7 = 0.00794 [lb/yd]

[yd/shift] 14074.34

No. of [gr/yd] of each sliver = [lb/yd] x 7000 = 55.55 [gr/yd]

No. of [gr/yd] of 24 slivers = 55.55 x 24 = 1333.33 [gr/yd]

Now total production;

P = π DN x 60 x lap ct.(gr/yd) x η x no. of m/c [lb/hr] 36 7000

P = π x 16” x 30 x 60 x 1333.33 (gr/yd) x 8 x 0.7 x 1 [lb/shift] 36 7000

= 2680 [lb/shift] ------Ans

2—Ribbon Lap M/c

Q:6-Calculate the production of a ribbon lap m/c in 8 hours at 70% efficiency if the speed of 16” dia. lap drum is 48 rpm and hank of ribbon lap is 0.0119.

Feeding sliver count = 0.0119 Hk

Lap roller speed, N = 48 rpm

Lap roller dia., D = 16”

Efficiency, η = 70% = 0.7

P [lb/shift] = ?

Solution:-

No. of yards of lap delivered in 1 shift at 70% efficiency

= π DN x 60 x 8 hr x η [yd/shift]36

= π x 16” x 48 x 60 x 8 hr x 0.7 [yd/shift]36

= 22519 [yd/shift]

No. of pounds of lap delivered in 1 shift at 70% efficiency
= [yd/shift] = 22519 yd x lb = 2253.7 [lb/shift]
sliver count shift 0.0119 x 840 yd
Ans

5. Production of Comber

P = f (π DN) x 60 x sliver ct.(gr/yd) x η x N x no. of x no. of x 1 - w
36 7000 heads m/c 100
[lb/hr]
Here,
f = feeding rate
N = nips/ min of m/c
w = waste %age

Q:7–The cylinder of a 6 head comber is running at a speed of 100 nips per minute and each nip feeds 0.25" lap. The hank of lap is 0.0166. calculate the production of comber in 8 hours at 70% efficiency and 12% waste:-

Feeding rate = 0.25" /min
Count of lap = 0.0166 Hk
= 0.0166 x 840 (yd/lb)
= 1 / 13.94 (lb/yd) = 0.0717 (lb/yd)
No. of heads = 6
Nips/min = 100
No. of m/c = 1
Efficiency = 70% = 0.7
Waste %age = 12%

Solution:-
P = f (π DN) x 60 x sliver ct.(gr/yd) x η x N x no. of x no. of x 1 - w .
36 7000 heads m/c 100 [lb/hr]
P = 0.25 x 60 x 0.0717 (lb/yd) x 0.7 x 100 x 6 x 1 x 1 - 12 .36 100 [lb/hr]
= 11.04 [lb/hr]
= 88.33 [lb/shift] ------Ans

6. Production of Simplex

$$P = \frac{\pi D N \times 60}{36} \times \frac{\text{roving ct. (gr/yd)}}{7000} \times \eta \times \text{no. of spindles [lb/hr]}$$

Also,

$$P = \frac{\pi D N \times 60}{36} \times \frac{\text{roving ct. (gr/yd)}}{7000} \times \eta \text{ [lb/hr/spindle]}$$

This formula is used when the production of a single spindle is concerned.

Q:8-A simplex frame working at 80% efficiency prepares a full doff in 3. hours. The wt. of roving on full bobbin is 3 lb and 4 oz. The hank of roving is 1.0. Calculate the production of a frame of two doffs in hanks and the speed of the front roller of 1⅛ " diameter:-

(When the required production of a m/c on its output package is complete, it is said to be one doff and the process of replacing these full packages with the empty ones to get further output is known as doffing)

Efficiency, η = 80% = 80/100 = 0.8

Time to complete one doff = 3. hr

Wt. of roving on full bobbin = 3 lb + 4 oz

= 3 lb + 4/16 lb

= 3.25 lb

Hank of roving = 1.0

Dia. of Front Roller, D = 1⅛ " = 1.125"

Production of a frame of two doffs, P_2 = ?

Speed (rpm) of Front Roller, N = ?

Solution:-

1 doff (3. hr) makes a bobbin of roving wt------------------------3.25 lb

2 doffs (7 hr) make a bobbin of roving wt-------------------------3.25 x 2

= 6.5 lb

Production, P_2 (Hk/ 2 doffs) = wt. of 2 doffs (lb) x Hk of roving

= 6.5 x 1

= 6.5 Hk in 7 hr---------Ans

Production, P (Hk/hr) = 6.5 / 7 = 0.93 Hk / hr

On simplex we have,

P (Hk/hr) = ´ DN x 60 x no. of spindles x η

36 840 roving Hk

N = P x 36 x 840 x 1 x 1

´D 60 η 1

= 0.93 x 36 x 840 x 1 .

1.125 ´ 60 0.8

= 165.4 rpm------Ans

7. Production of Ring frame

P = π DN x 60 x 16 x 8 x η [oz/shift/spindle]

TPI x 36 840 x ct.

P = P [oz/shift/spindle] x no. of spindles x no. of frames [oz/shift]

DERIVATION

Let D = dia. of front rollers (in inches)

N = rpm of front rollers

η = efficiency of m/c

Production = surface speed of front rollers x yarn ct. (wt/l) or,

= delivery speed of F.R. x yarn (oz/yard) [oz/hr]

Now let us calculate the delivery speed of F.R.

On a ring frame,

TPI = spindle speed (rpm)

F.R. delivery (in/min)

F.R. delivery = spindle speed (rpm) [in/min] TPI

F.R. delivery = sp. speed x 60 [yd/hr] TPI x 36

Now substituting this value in the production formula,

P [oz/yd] = sp. speed x 60 [yd/hr] x yarn ct. [oz/yd] TPI x 36

= sp. speed x 60 [yd/hr] x yarn ct. [oz/yd] x η [oz/yd] TPI x 36

As 1 shift = 8hr and this is the calculation for a single spindle so,

P [OPS] = sp. speed x 60 x 8 x yarn ct. [oz/yd] x η[oz/shift/spindle]TPI x 36

However, in some cases the English count is given instead of oz/yd of yarn.

For that purpose, let us make some changes in the above formula,

= sp. speed x 60 x 8 x η [lb/shift/spindle]TPI x 36 840 x ct.

= sp. speed x 60 x 8 x 16 x η [oz/shift/spindle] TPI x 36 840 x ct.

This formula helps to calculate the production of one spindle. For the production of a full ring frame,

P [oz/shift] = P [oz/shift/spindle] x no. of spindles [oz/shift]

The no. of spindles in one ring frame is 480. This is a fixed value and can be used when spindle capacity of the ring frame is not mentioned. Also, if the production of more than one ring frames is to be calculated, then

P [oz/shift] = P [oz/shift/spindle] x 480 x no. of frames [oz/shift]

As 1 day = 3 shifts and 1 bag = 100 lb,

P [bag/day] = P [oz/shift/spindle] x 480 x 3 x no. of [bag/day]

16 x 100 frames

= 0.9 x P [oz/shift/spindle] x no. of frames [bags/day]

Q:9-Calculate the production of yarn in oz/spindle/shift on a ring frame if the spindle speed is 16000" /min, TM is 3.8, yarn is 30/1 and efficiency of the m/c is 93%:-

Yarn count = 30/1

Efficiency = 93% = 0.93

No. of spindles = 480

TM = 3.8

Solution:-

TPI = TM √ ct.

= 3.8 √ 30

= 20.78

Now,

P [OPS] = sp. speed x 60 x 8 x 16 x η [oz/shift/spindle]

TPI x 36 840 x ct.

`= 16000 x 60 x 8 x 16 x 0.93 [oz/shift/spindle]

20.78 x 36 840 x 30

= 6.06 [oz/shift/spindle]

P [bag/day] = P [oz/shift/spindle] x 480 x 3 x no. of [bag/day]

16 x 100 frames

= 6.06 x 480 x 3 x 1 [bag/day]

16 x 100

= 5.45 [bag/day] --------Ans